SQIsign

(for humans)

Krijn Reijnders, COSIC, KU Leuven
Cloudflare, June 19, 2025

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SQIsign

~~(for humans)~~

(for people that know a bit of ECC, ECDSA, and math, but not too much to see where I shove away technically challenging material)

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SQIsign

(for nerds)

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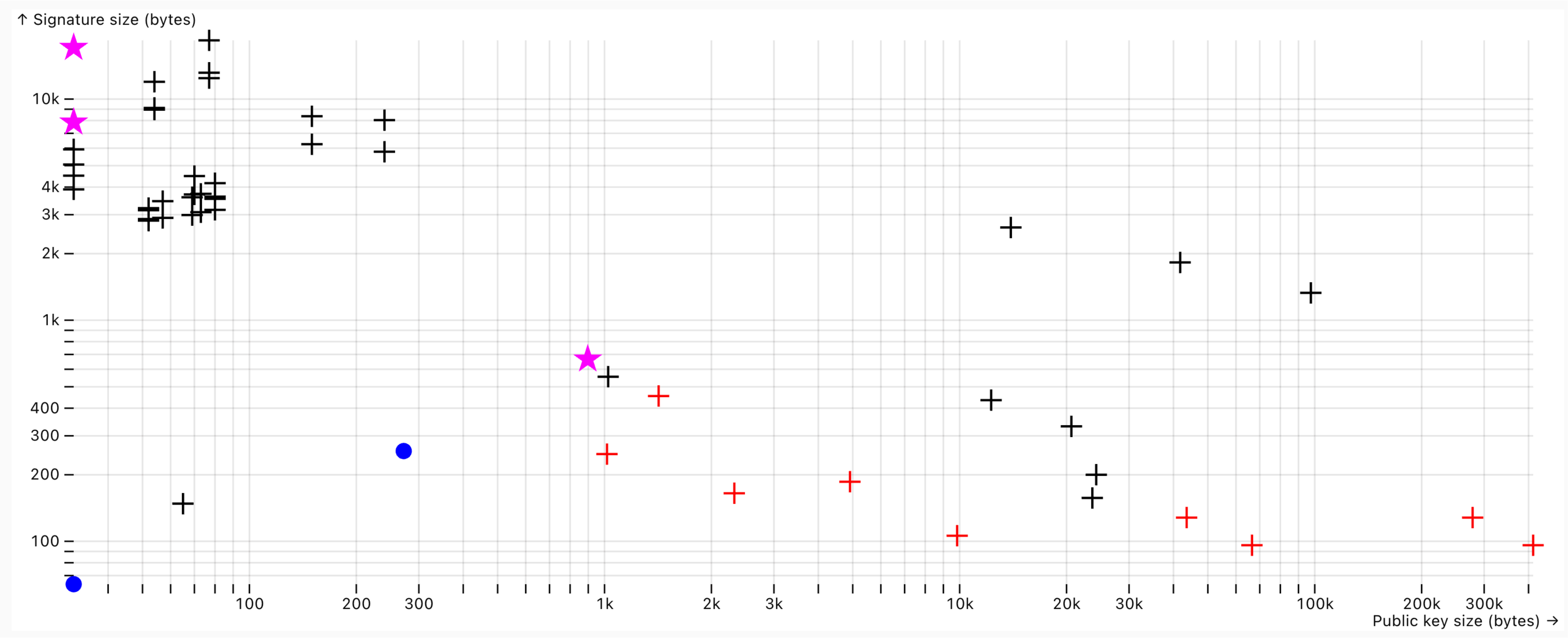
↑ Signature size (bytes)
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Public key size (bytes) →


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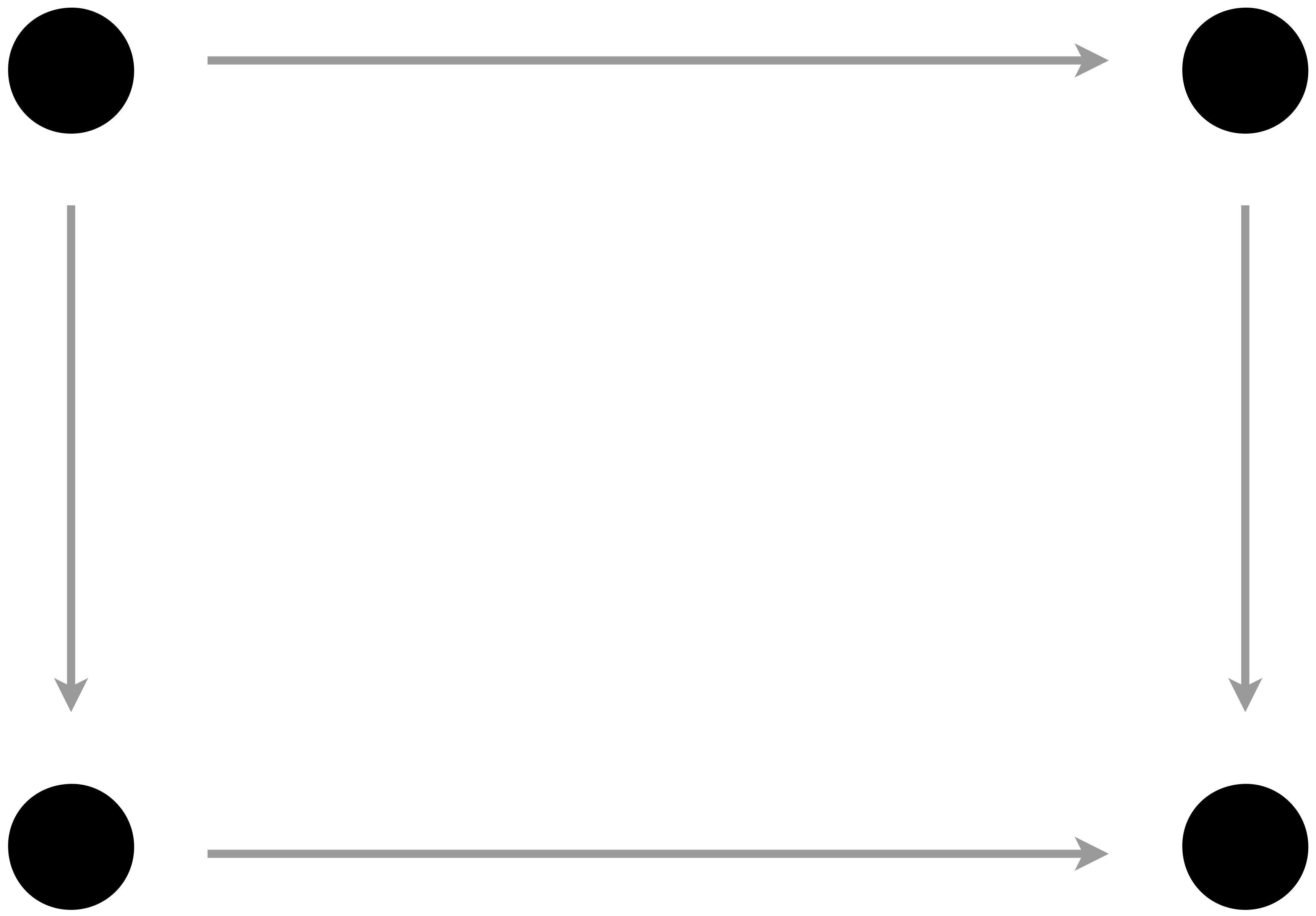
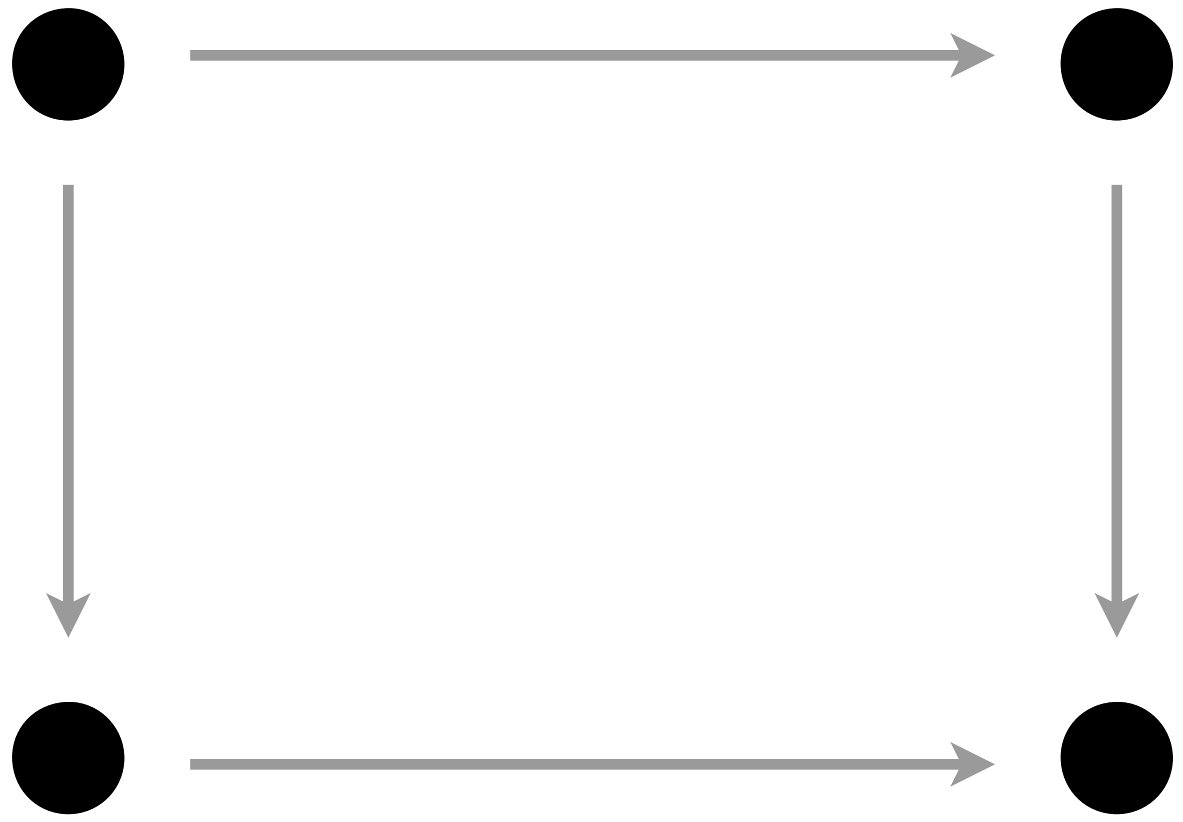
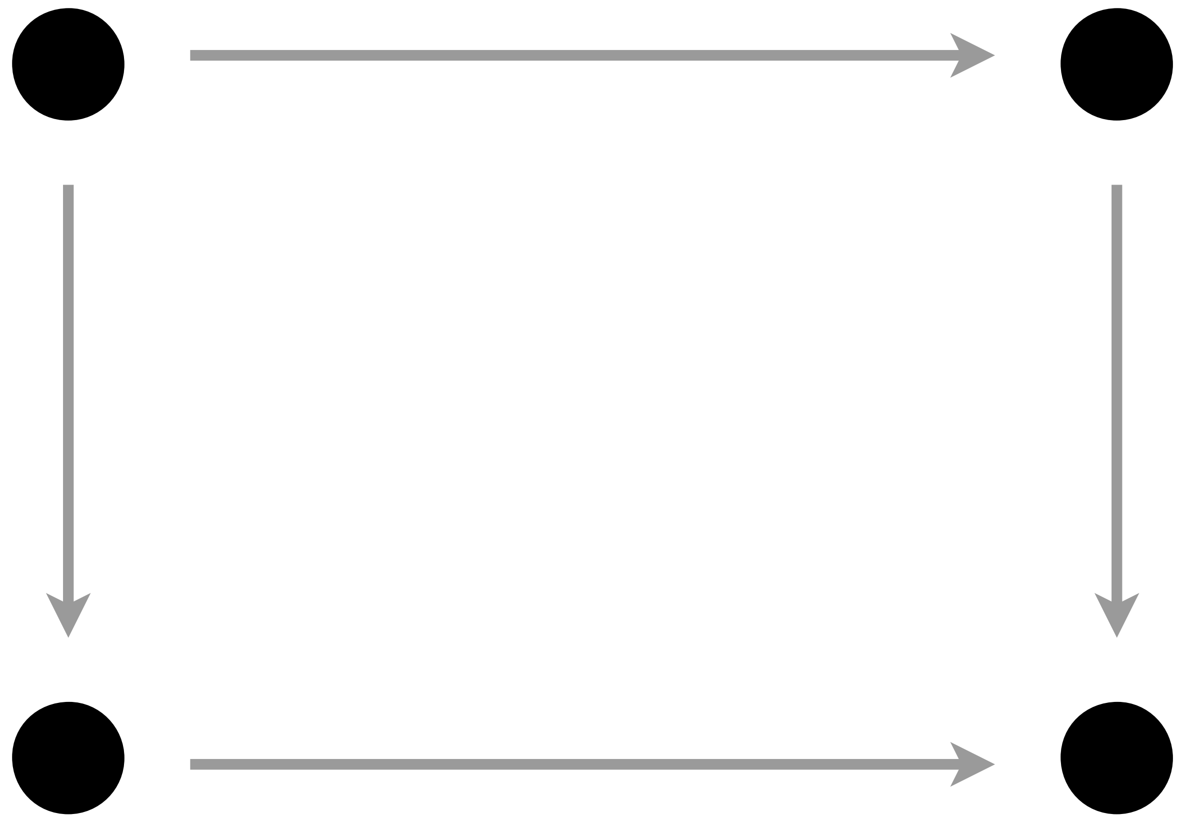
SQIsign!
OTHER PQ SCHEMES





setup

1. Take your favourite elliptic curve $\mathscr{E}$



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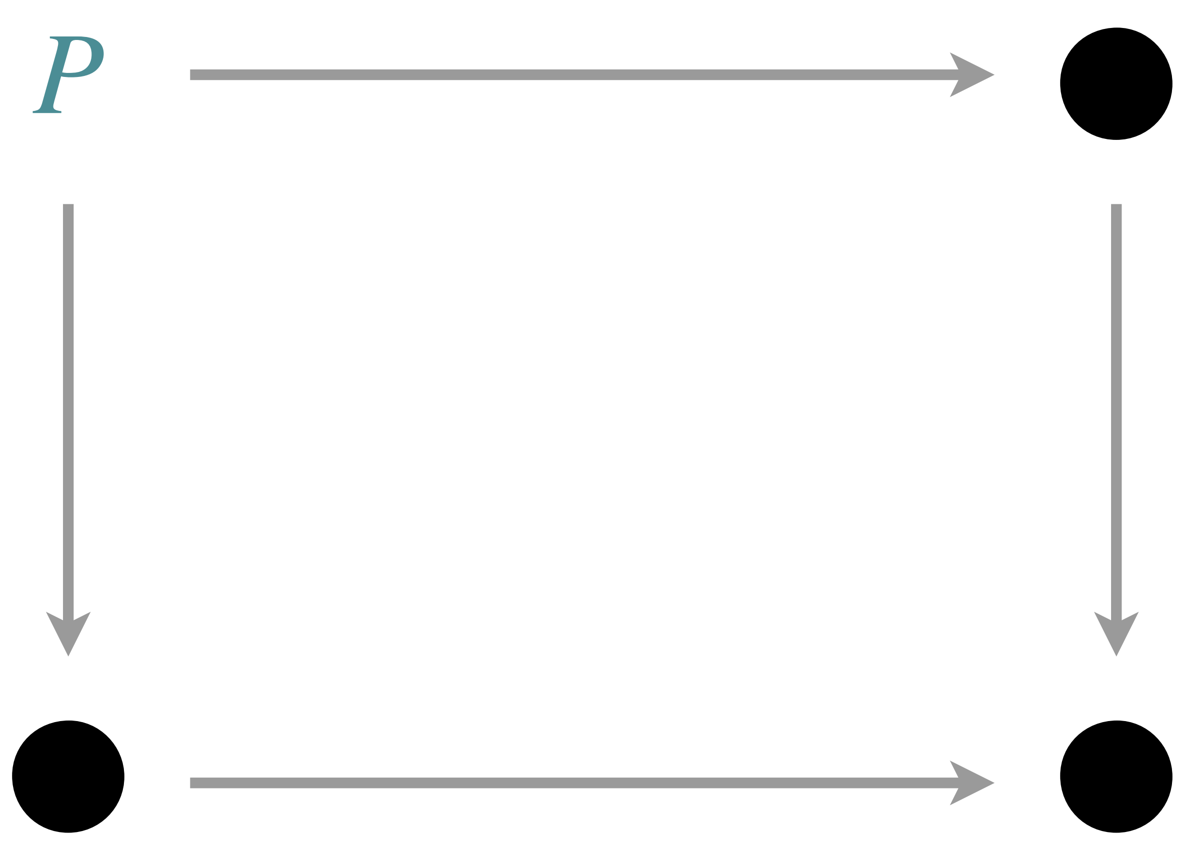
1. Take your favourite elliptic curve $\mathscr{E}$
2. Take prime-order subgroup $G \subset \mathscr{E}(\mathbb{F}_p)$ with generator P



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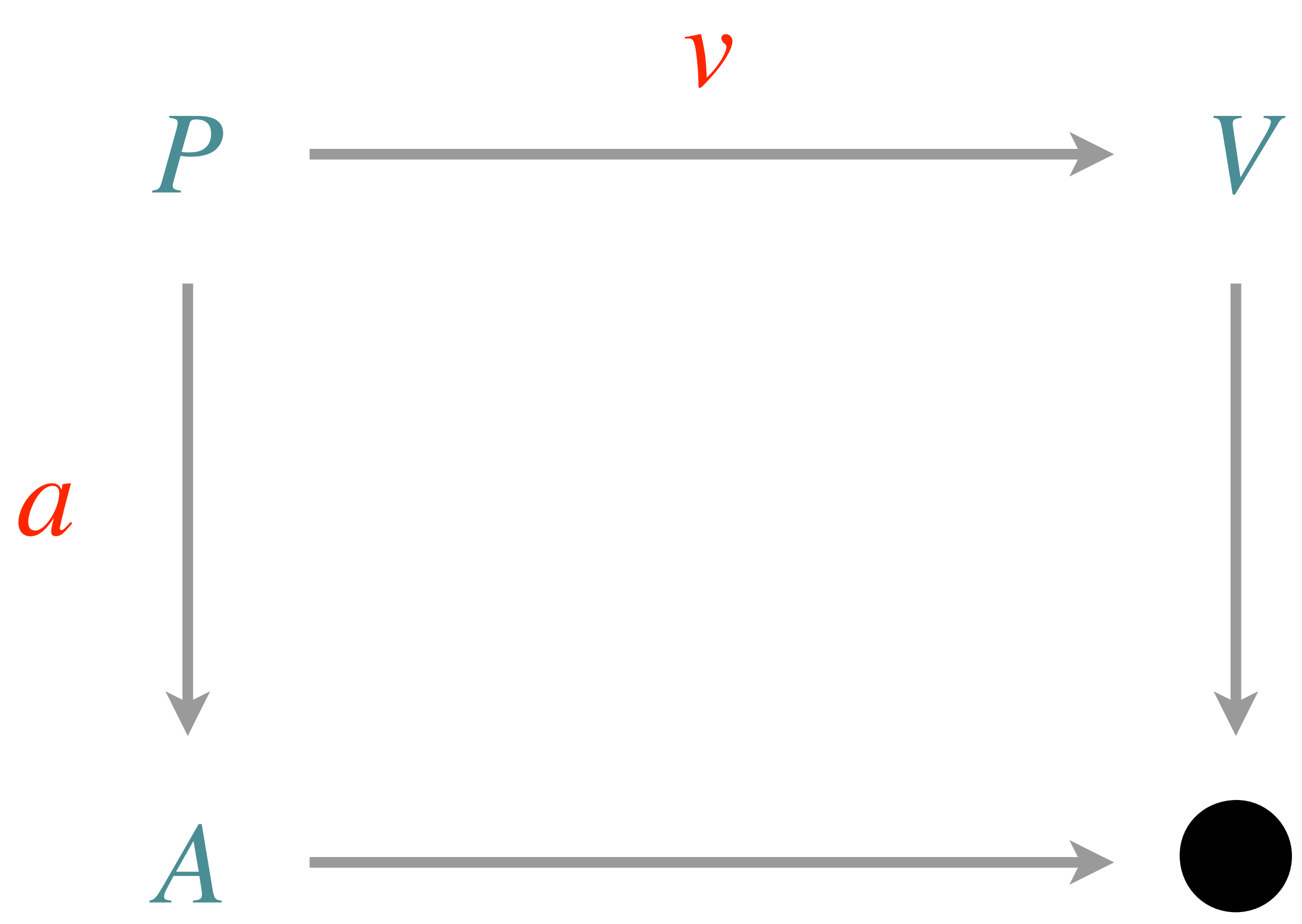

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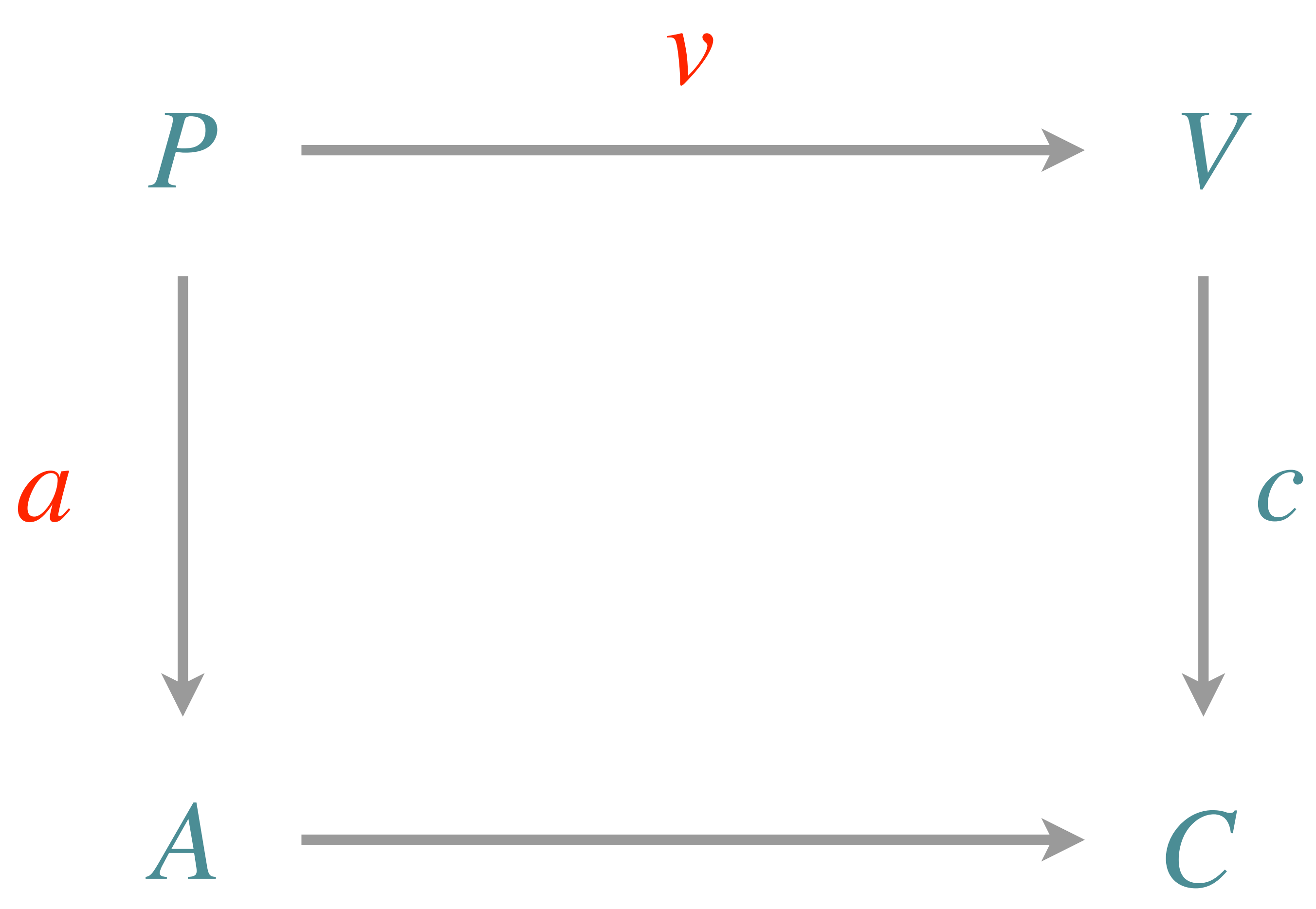

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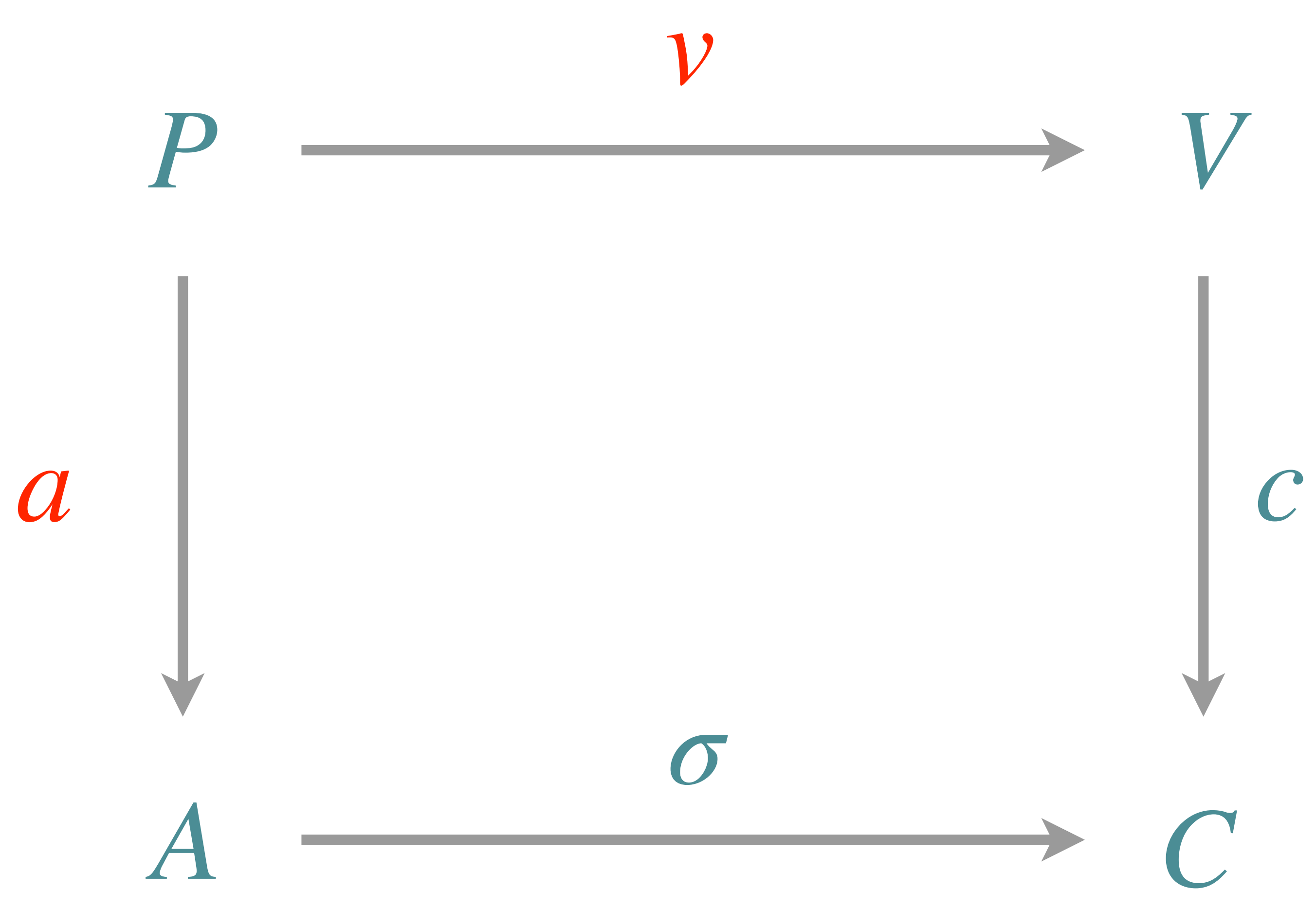
3. Return $\sigma = \dfrac{v \cdot c}{a} \in \mathbb{Z}_q^\times$ as response



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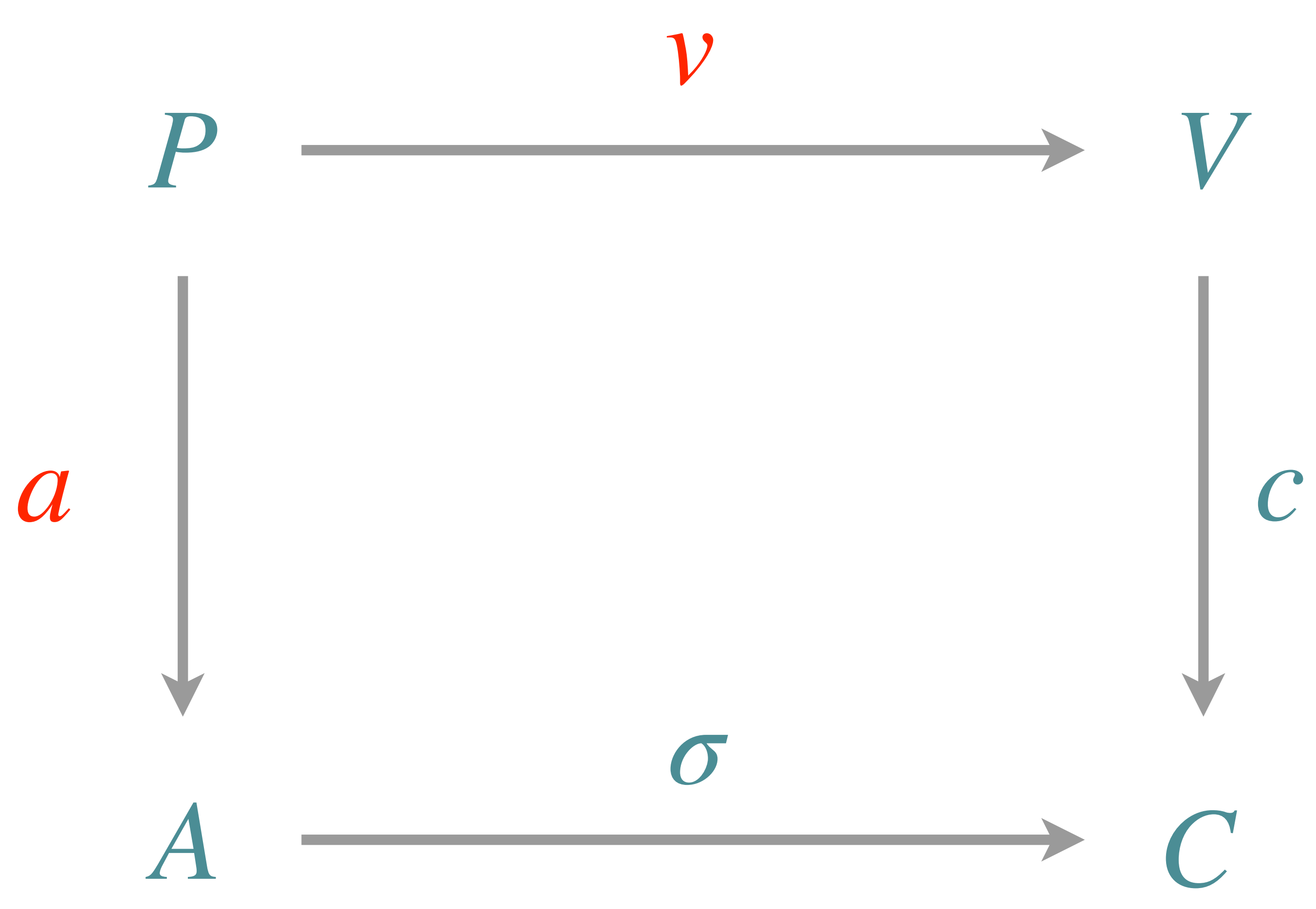

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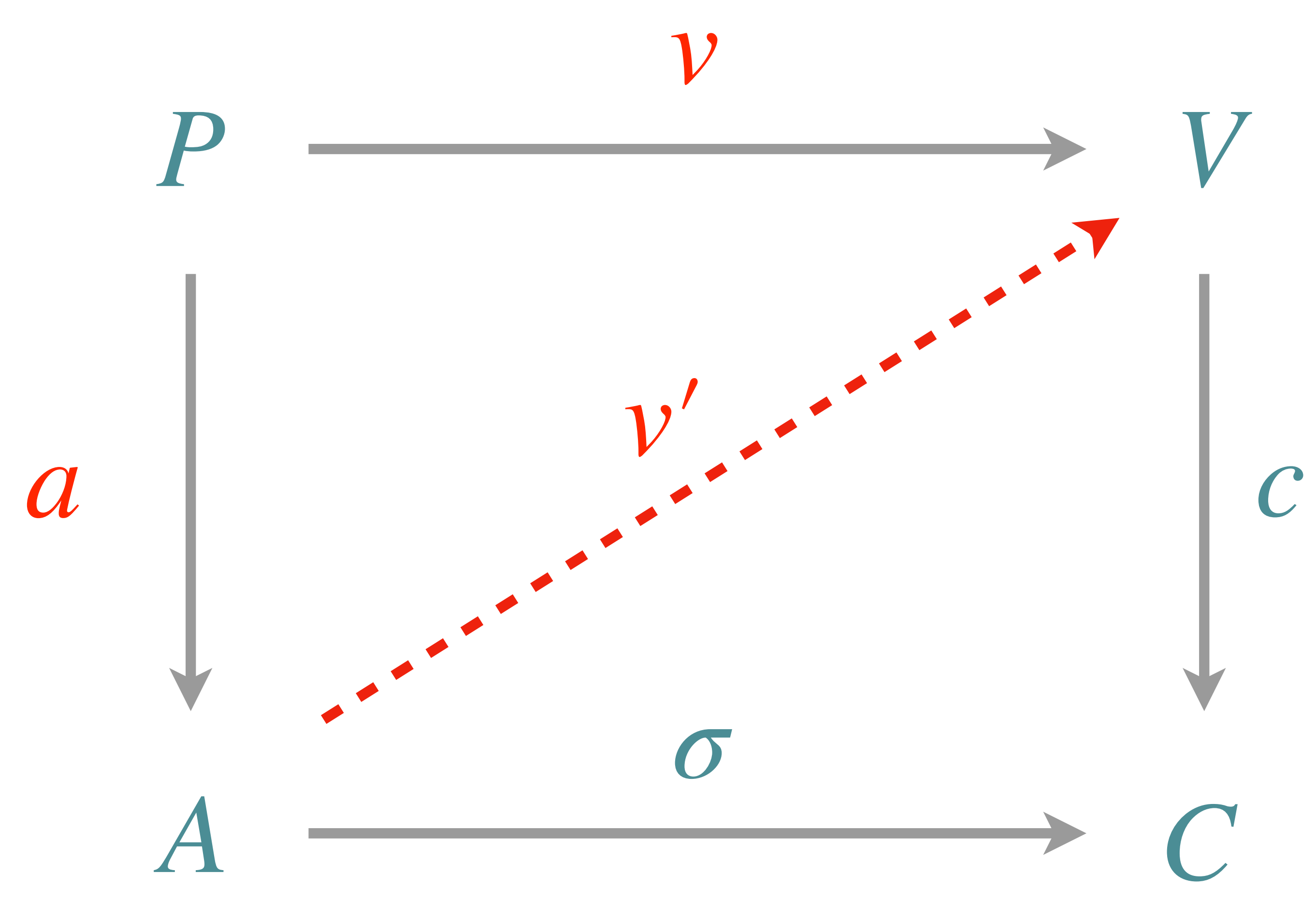
1. Take random v',
   commit to $V' = [v']A$
2. On challenge c,
   return $\sigma = v' \cdot c$



Our plan for today

1

Making the square work…

$$\mathscr{E} \xrightarrow{\varphi} \mathscr{E}'$$

with isogenies!

2

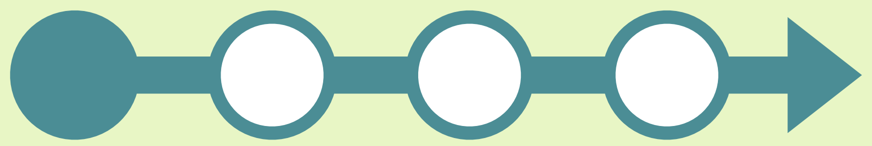
Decomposing the square

$$\mathrm{End}(\mathscr{E}) \xrightarrow{\sim} \mathscr{O}$$

with quaternions!

3

SQIsign, SQIsignHD



SQIsign2D, SQIsignXD…?

PART 1
The Square

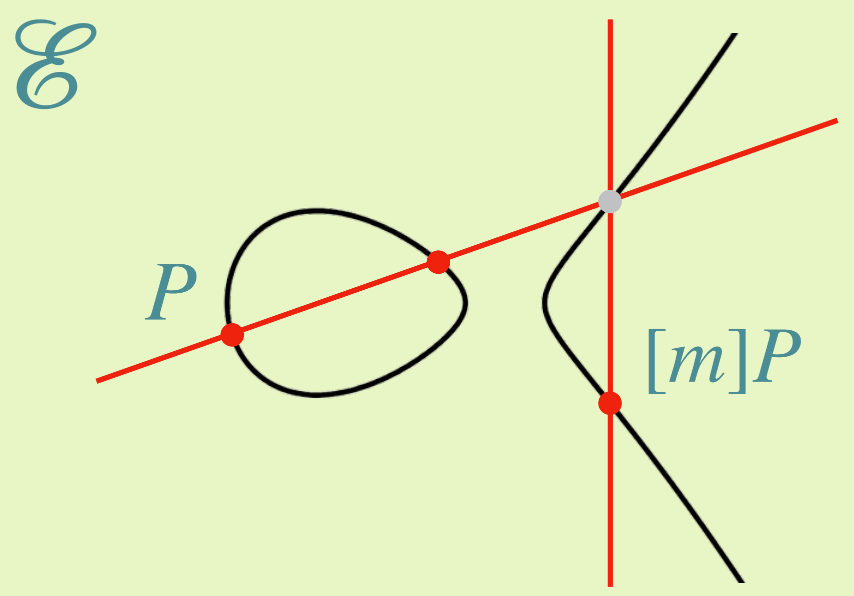
From ECC World to Isogeny World

ECC

- work on single 'nice' curve $\mathscr{E}$

$$\mathscr{E} : y^2 = x^3 + Ax^2 + x, \qquad A \in \mathbb{F}_p$$

- take a starting point P and perform scalar multiplications $[m] \in \mathbb{Z}_q^\times$



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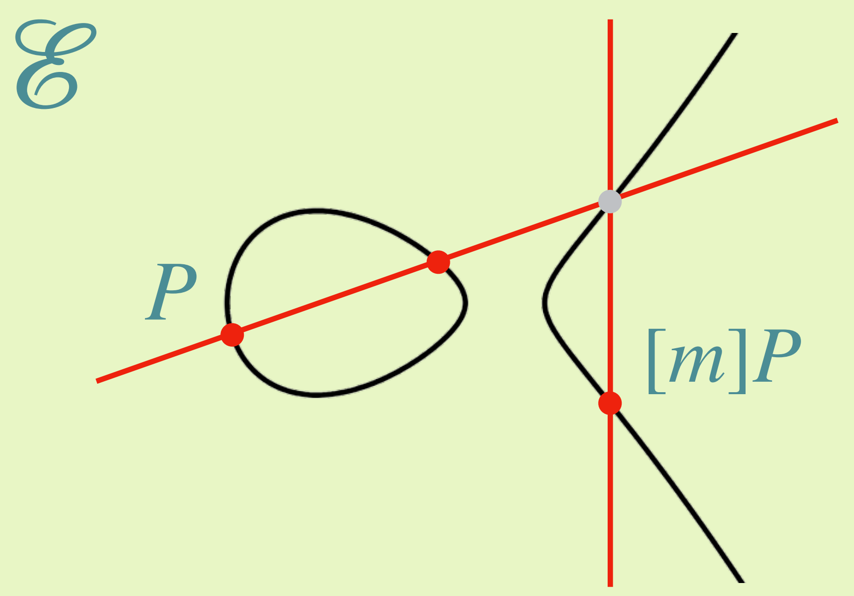
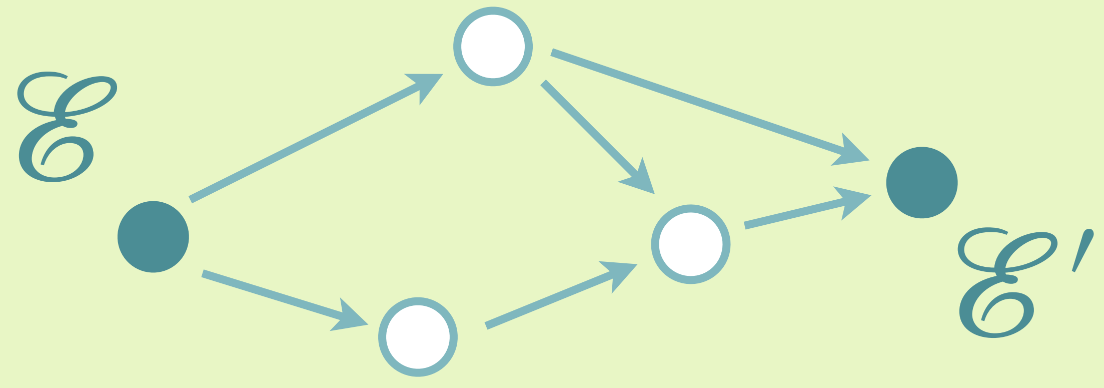


ISOGENY

- work in the whole world of curves!
- use 'nice' maps between curves, we call an **isogeny**

$$\varphi : \mathscr{E} \to \mathscr{E}'$$

- take a starting curve $\mathscr{E}_0$ and perform isogenies φ, ψ, θ



You “know” isogenies already!

in general

- map from $\mathscr{E}$ to itself is **endomorphism**
- simplest examples $[m] : P \mapsto [m]P$
- also easy $\pi : (x, y) \mapsto (x^p, y^p)$

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- we write $\mathrm{End}(\mathscr{E})$ for this **ring of endom**.

*this is not the formal definition

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a concrete example

$$\mathscr{E} : y^2 = x^3 + x \xrightarrow{\varphi} \mathscr{E}' : y^2 = x^3 + 5$$

$$(x, y) \mapsto \left(\frac{x^3 + x^2 + x + 2}{(x-5)^2}, \frac{y \cdot (x^3 - 4x^2 + 2)}{(x-5)^3} \right)$$

over $\mathbb{F}_{11}$

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hard problems

1 Given $\mathscr{E}$ and $\mathscr{E}'$, find an isogeny $\varphi : \mathscr{E} \to \mathscr{E}'$

2 Given a random $\mathscr{E}$, find a ‘funky’ endom. $\vartheta : \mathscr{E} \to \mathscr{E}$

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↔ *equivalent!*

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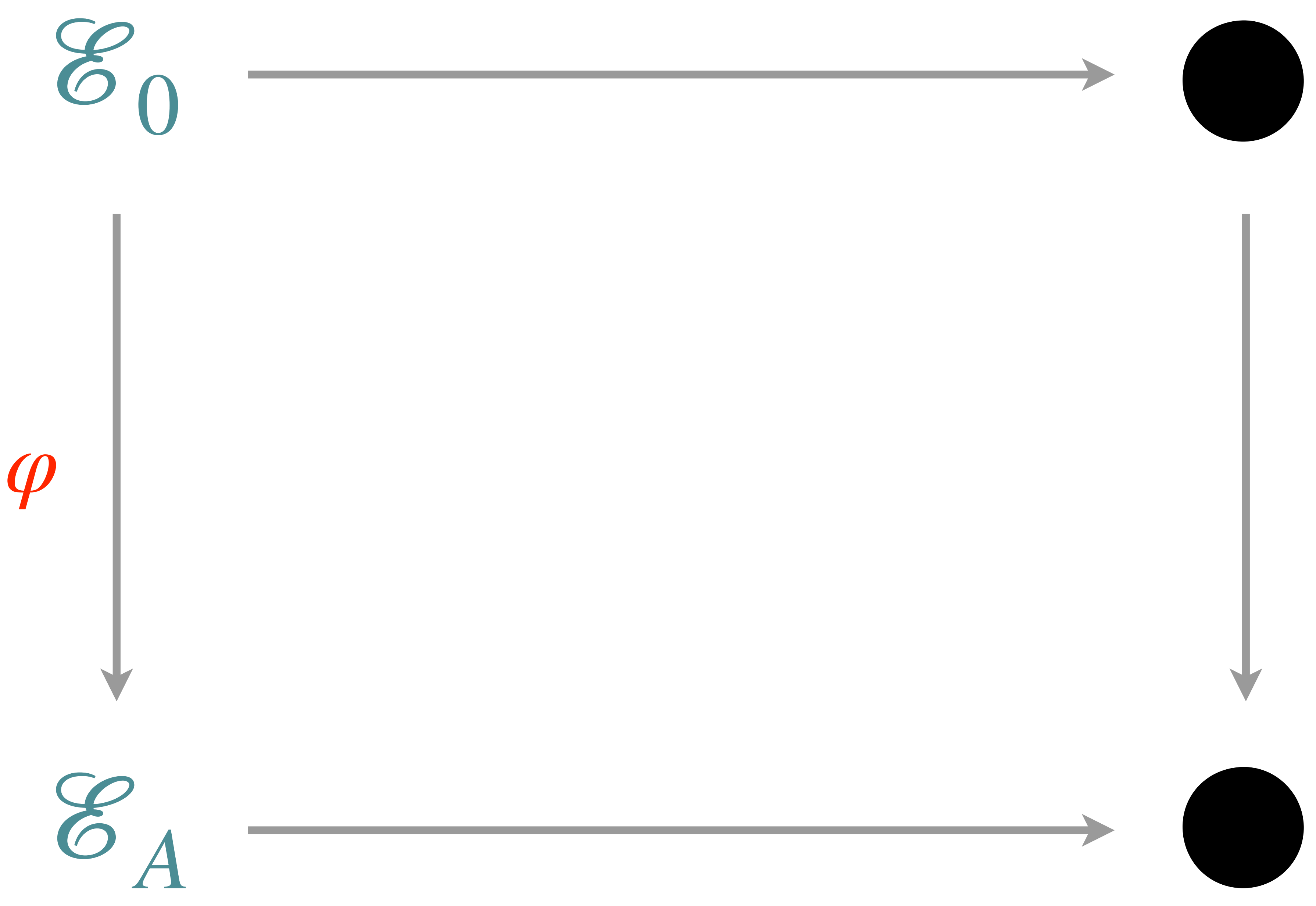

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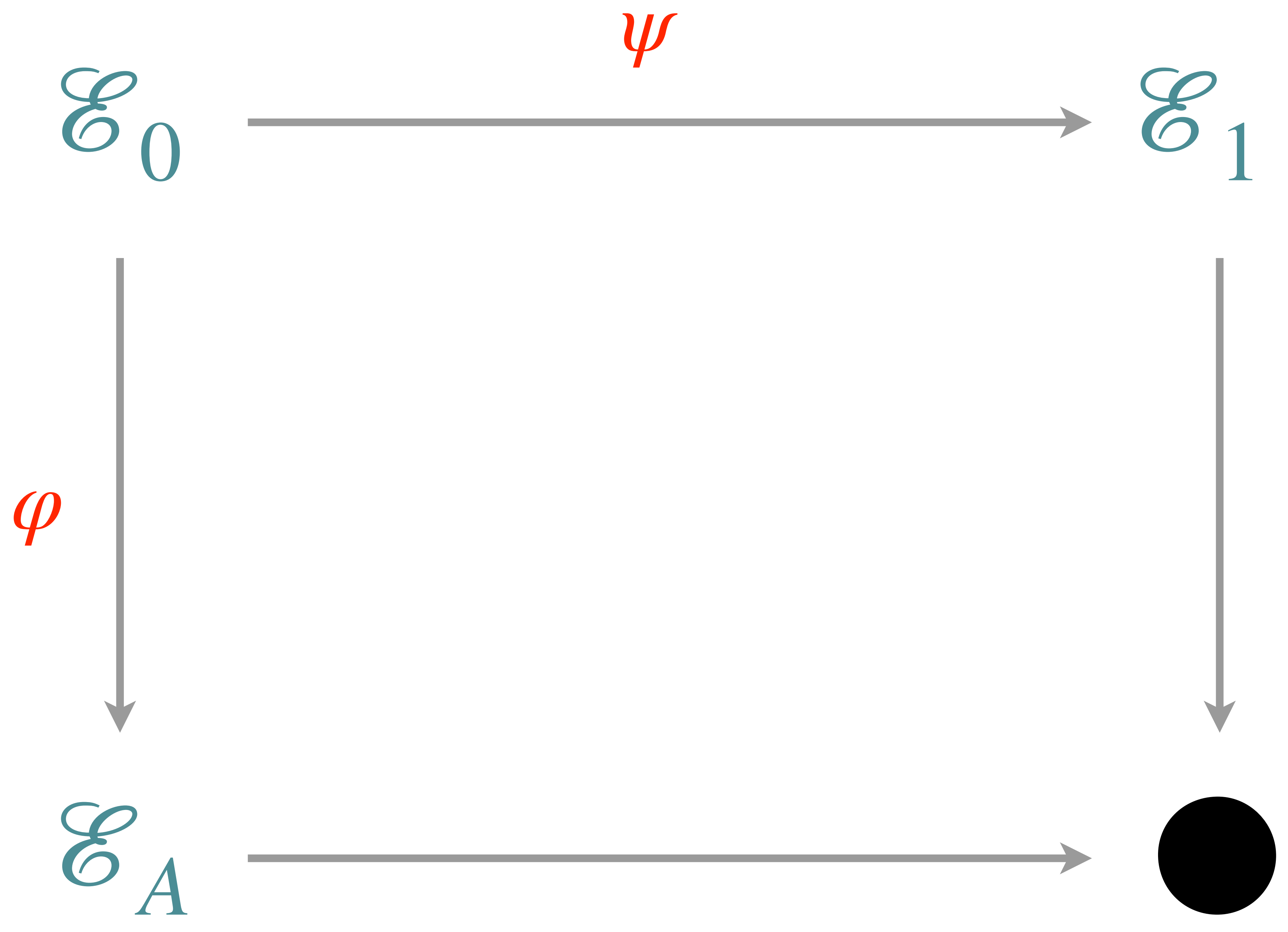

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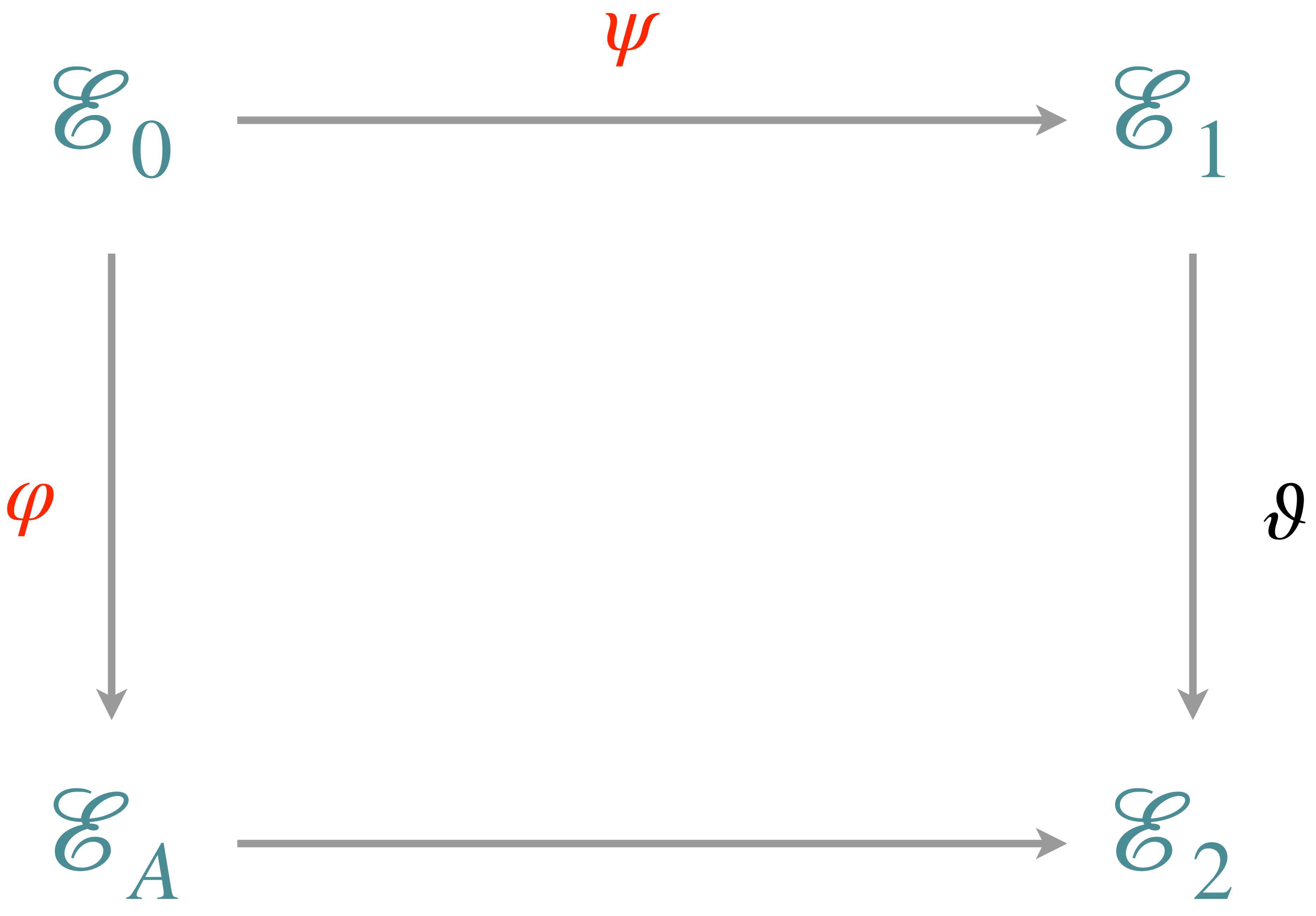

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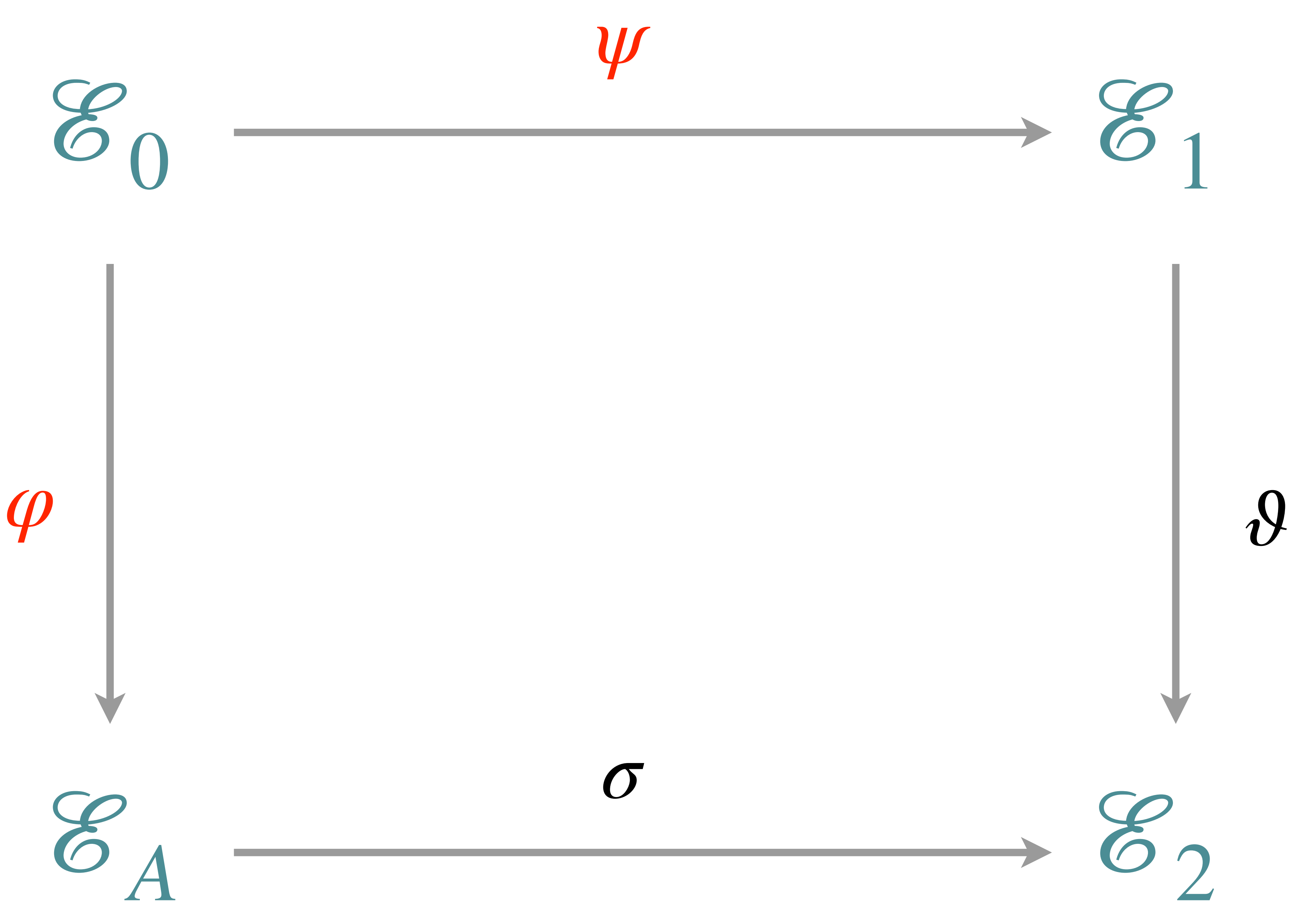

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KEY DIFFERENCE!

1. **DON'T** send $\sigma = \vartheta \circ \psi \circ \hat{\varphi}$, same issue as before
2. there are now *many* isogenies $\mathscr{E}_A \to \mathscr{E}_2$
3. set some requirement on σ, namely specific degree

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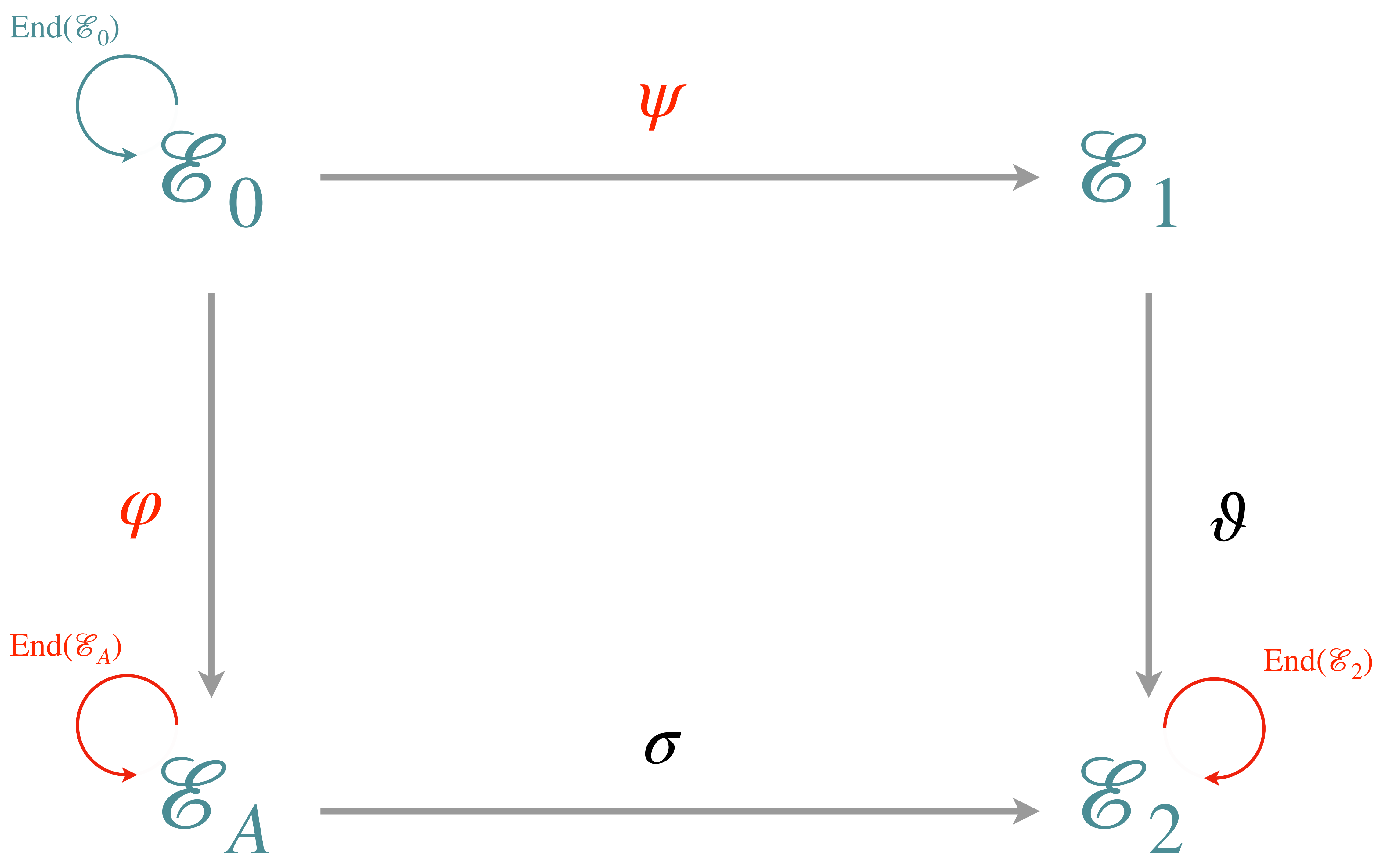

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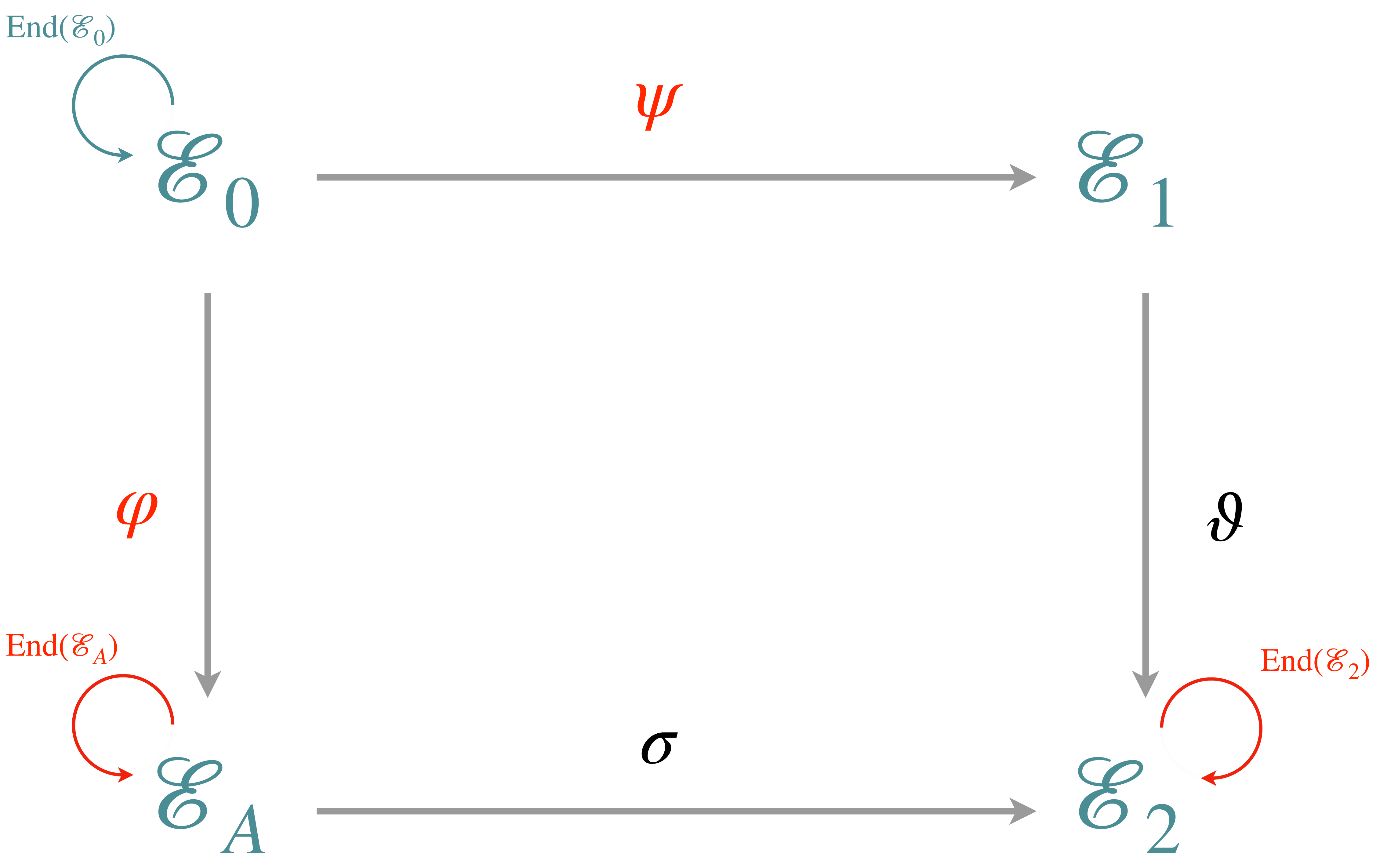

(magic)

🪄: if we know $\mathrm{End}(\mathscr{E}_A)$ and $\mathrm{End}(\mathscr{E}_2)$, we get σ ✨

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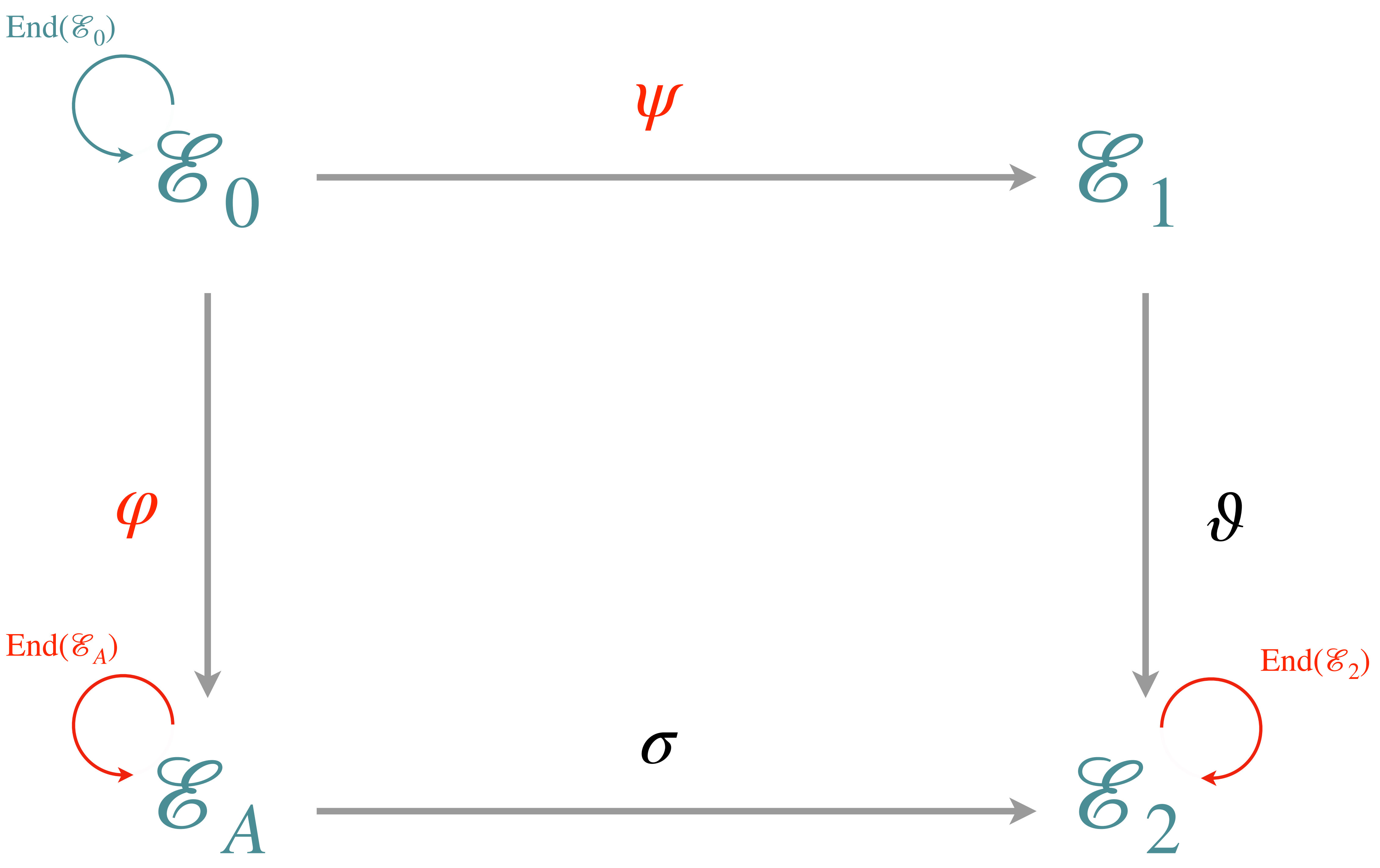

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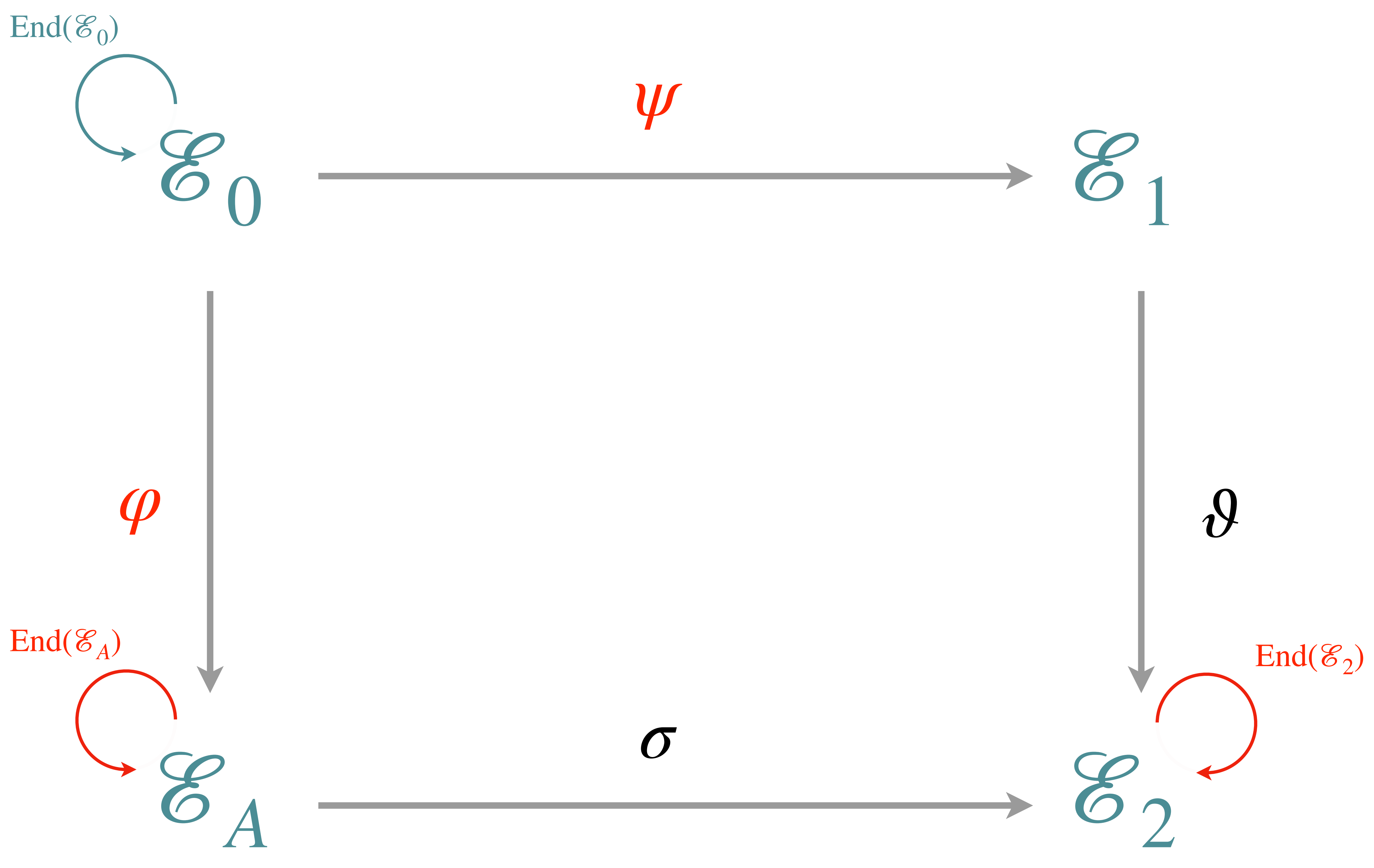

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1 We can only find such σ if we know $\mathrm{End}(\mathscr{E}_A)$ and $\mathrm{End}(\mathscr{E}_2)$

2 Given just random $\mathscr{E}$, impossible to find $\mathrm{End}(\mathscr{E})$

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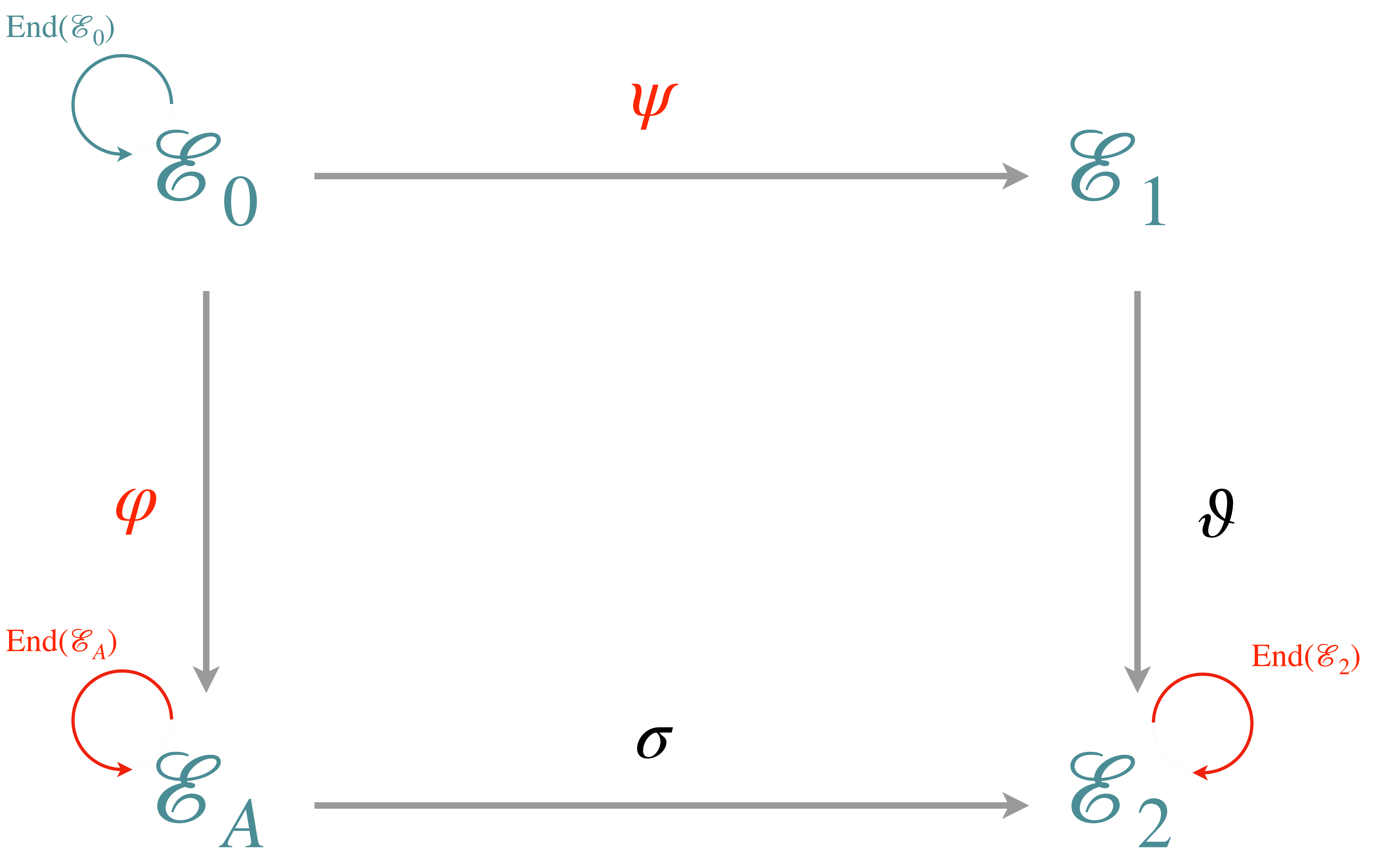

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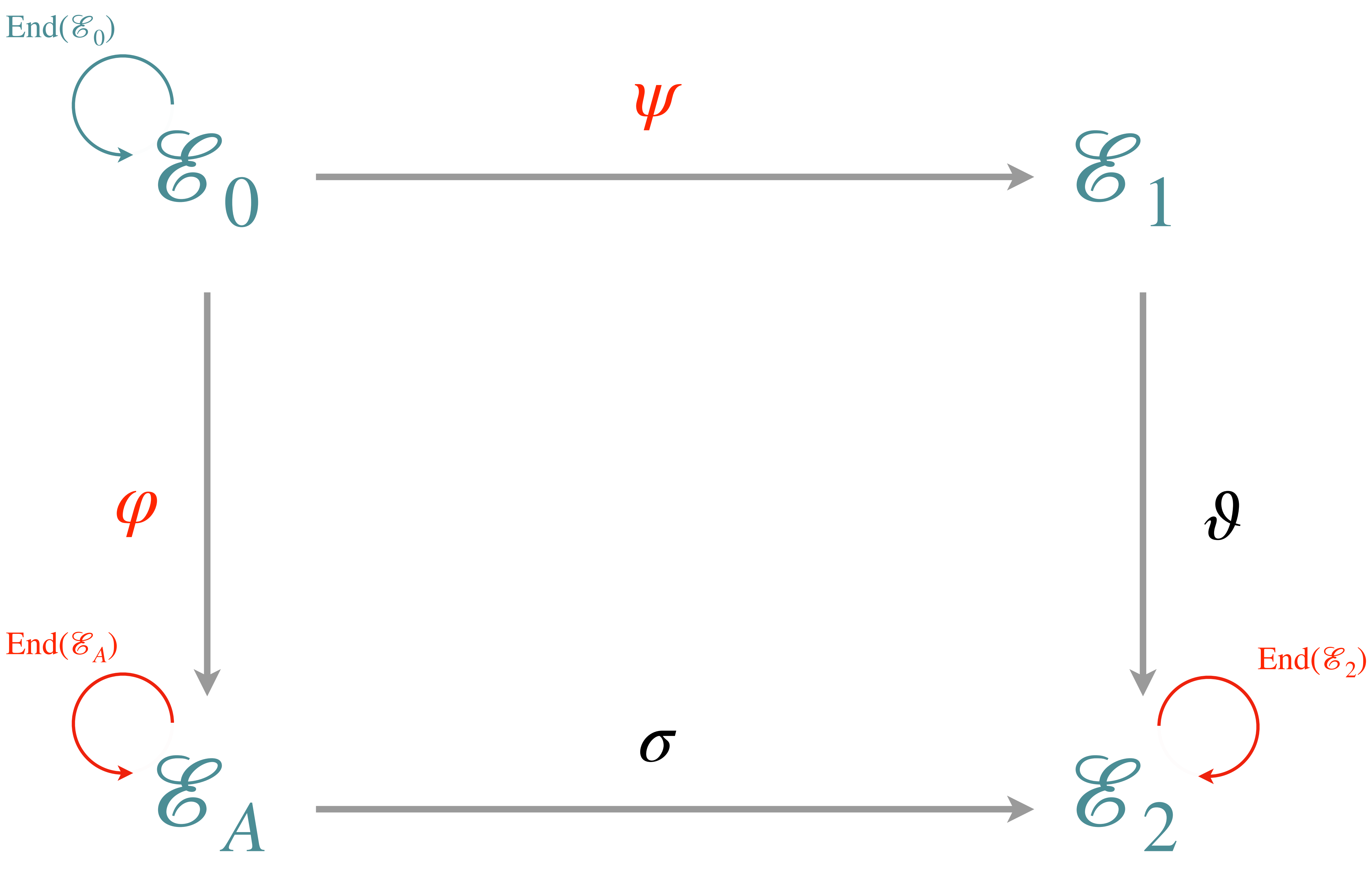

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! Returning σ of specific degree, proves knowledge of φ

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with quaternions!

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SQIsign, SQIsignHD

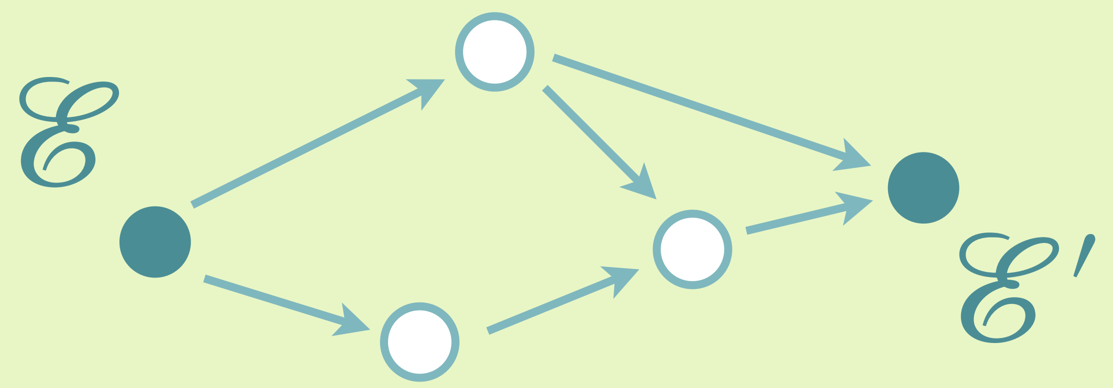
SQIsign2D, SQIsignXD…?

PART 2
Quaternions!

The Deuring correspondence transforms isogeny problems into quaternion problems

ISOGENY

- objects are **curves** $\mathscr{E}$, arrows are **isogenies** φ



- arrows from $\mathscr{E}$ to itself are **endomorphisms**, the ring of these, $\mathrm{End}(\mathscr{E})$ is very important for SQIsign, equal to the secret key
- if we know $\mathrm{End}(\mathscr{E})$ and $\mathrm{End}(\mathscr{E}')$, we can compute an arrow $\mathscr{E} \to \mathscr{E}'$

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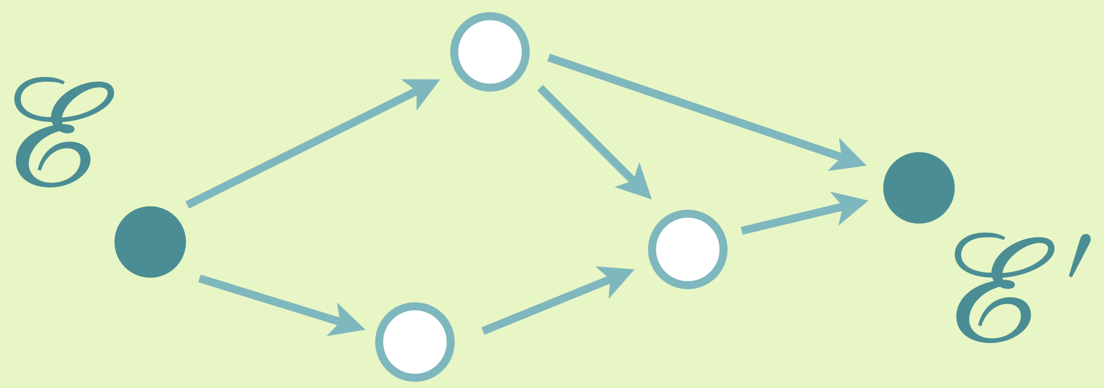

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QUATERNIONS

- a **quaternion** looks like $\beta = a + b \cdot \mathbf{i} + c \cdot \mathbf{j} + d \cdot \mathbf{k}$ where $a, b, c, d \in \mathbb{Q}$ and $\mathbf{i}^2 = -1, \mathbf{j}^2 = p, \mathbf{k} = \mathbf{i} \cdot \mathbf{j}$
- form a **non-commutative** algebra, like $\mathbb{C}$ on steroids

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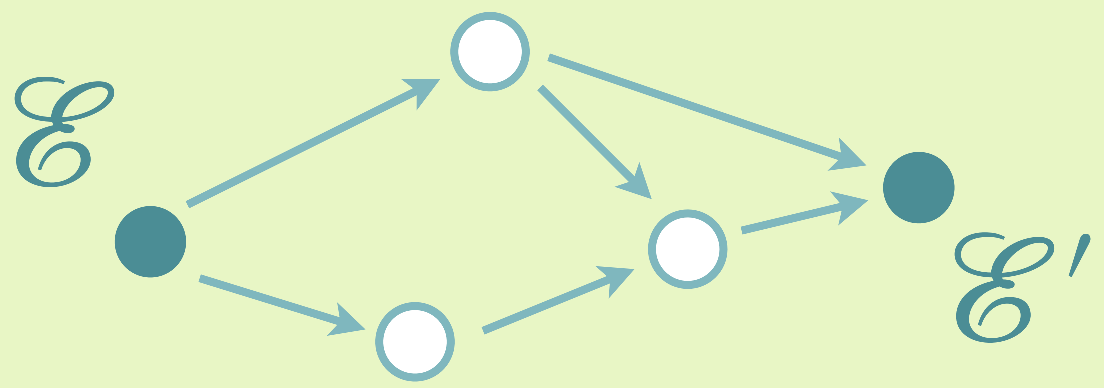

- arrows from $\mathscr{E}$ to itself are **endomorphisms**, the ring of these, $\mathrm{End}(\mathscr{E})$ is very important for SQIsign, equal to the secret key
- if we know $\mathrm{End}(\mathscr{E})$ and $\mathrm{End}(\mathscr{E}')$, we can compute an arrow $\mathscr{E} \to \mathscr{E}'$

QUATERNIONS

- a **quaternion** looks like $\beta = a + b \cdot \mathbf{i} + c \cdot \mathbf{j} + d \cdot \mathbf{k}$ where $a, b, c, d \in \mathbb{Q}$ and $\mathbf{i}^2 = -1$, $\mathbf{j}^2 = p$, $\mathbf{k} = \mathbf{i} \cdot \mathbf{j}$
- form a **non-commutative** algebra, like $\mathbb{C}$ on steroids
- precise mathematical details for this talk not necessary, just think “different mathematical world”
- objects are **maximal orders** $\mathcal{O}$, arrows are **ideals** I

PART 2
Quaternions!

The Deuring correspondence transforms isogeny problems into quaternion problems

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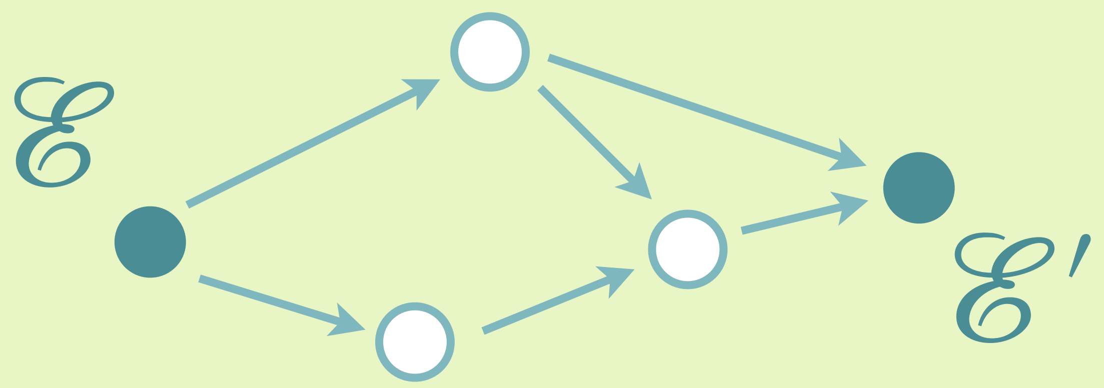

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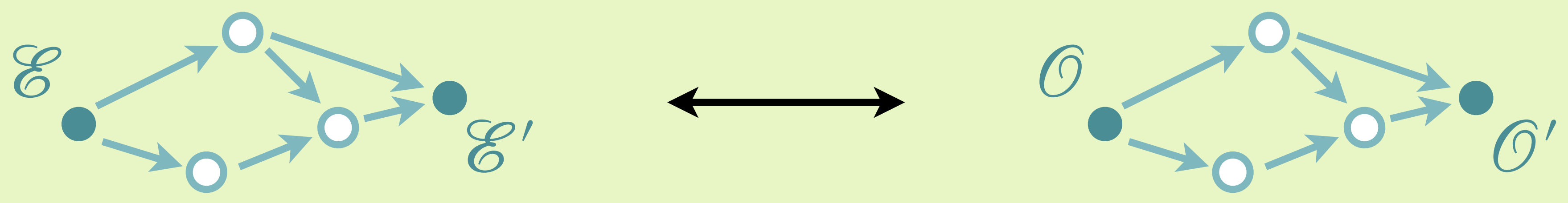

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PART 2
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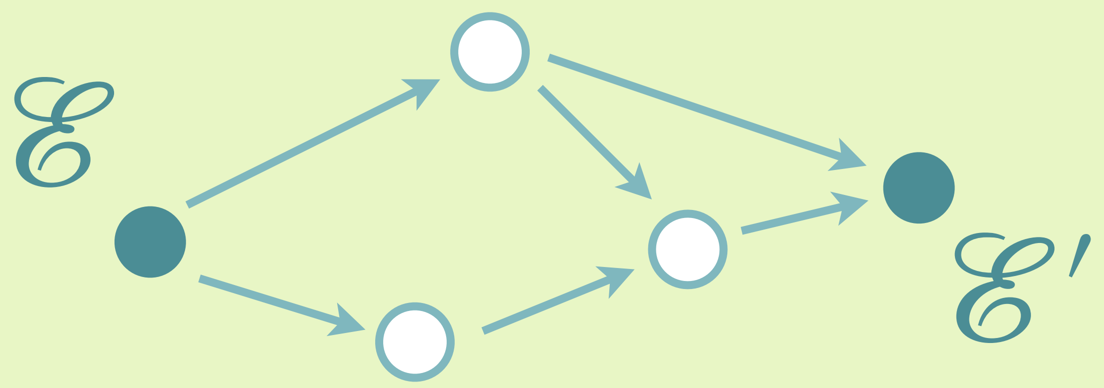

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Deuring

$\mathscr{E} \mapsto \mathrm{End}(\mathscr{E})$

$\mathcal{O} \cong \mathrm{End}(\mathscr{E})$

QUATERNIONS

- a **quaternion** looks like $\beta = a + b \cdot \mathbf{i} + c \cdot \mathbf{j} + d \cdot \mathbf{k}$ where $a, b, c, d \in \mathbb{Q}$ and $\mathbf{i}^2 = -1, \mathbf{j}^2 = p, \mathbf{k} = \mathbf{i} \cdot \mathbf{j}$
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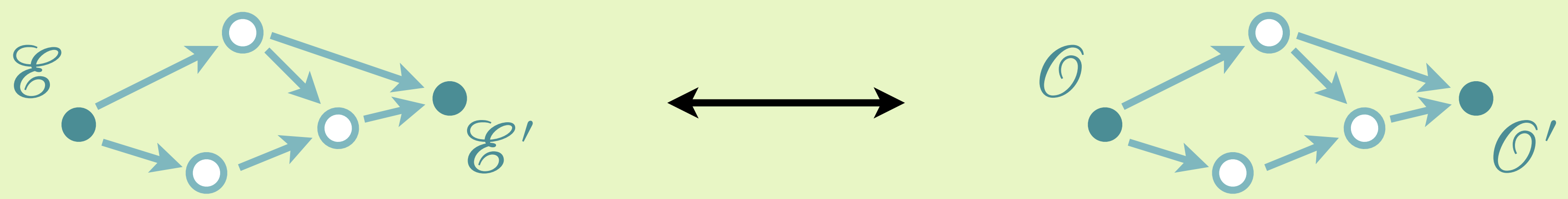

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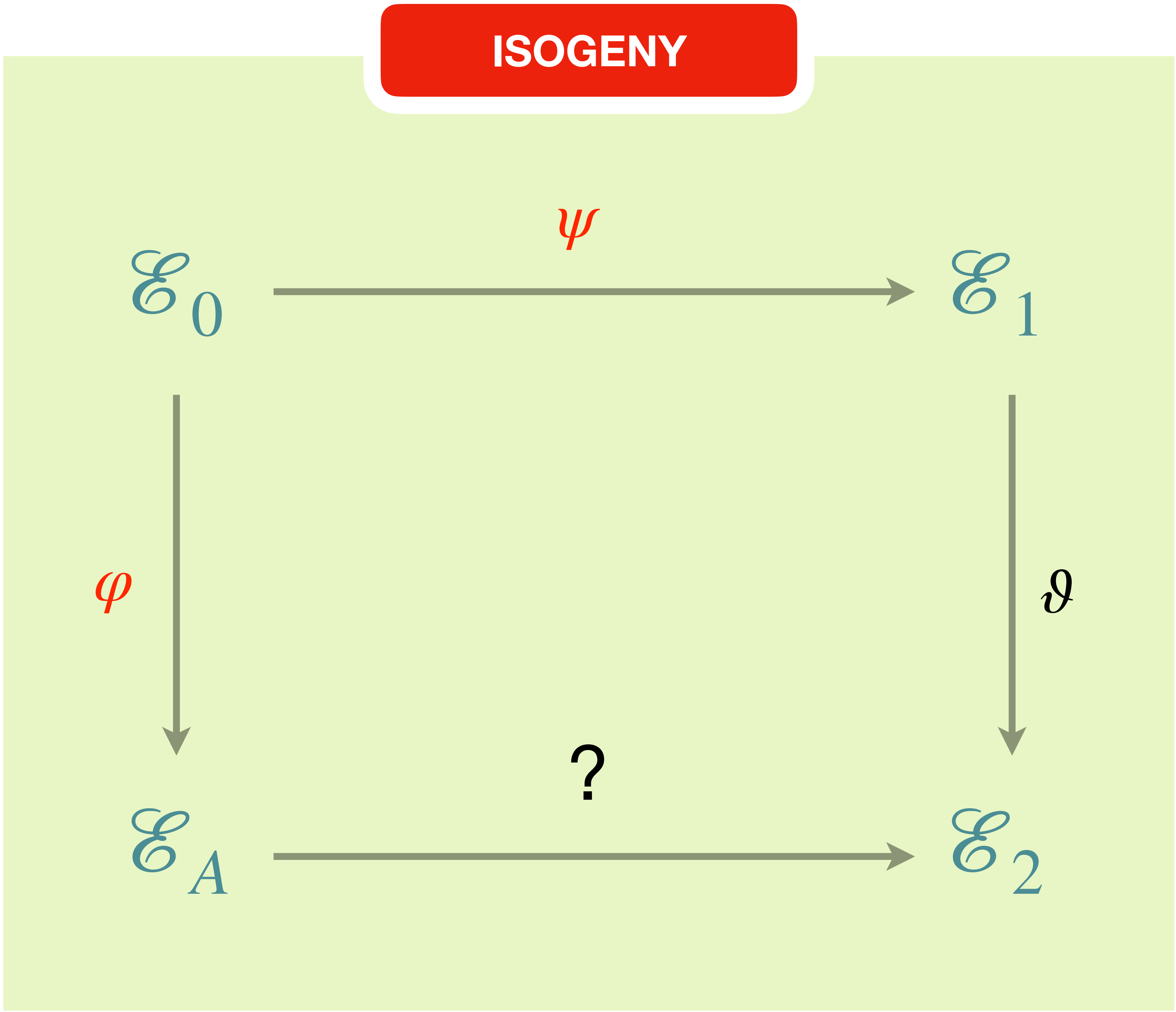
PART 2
Quaternions!

Translate the SQIsign square to the quaternion world: finding $\mathscr{E} \to \mathscr{E}'$ becomes advanced linear algebra



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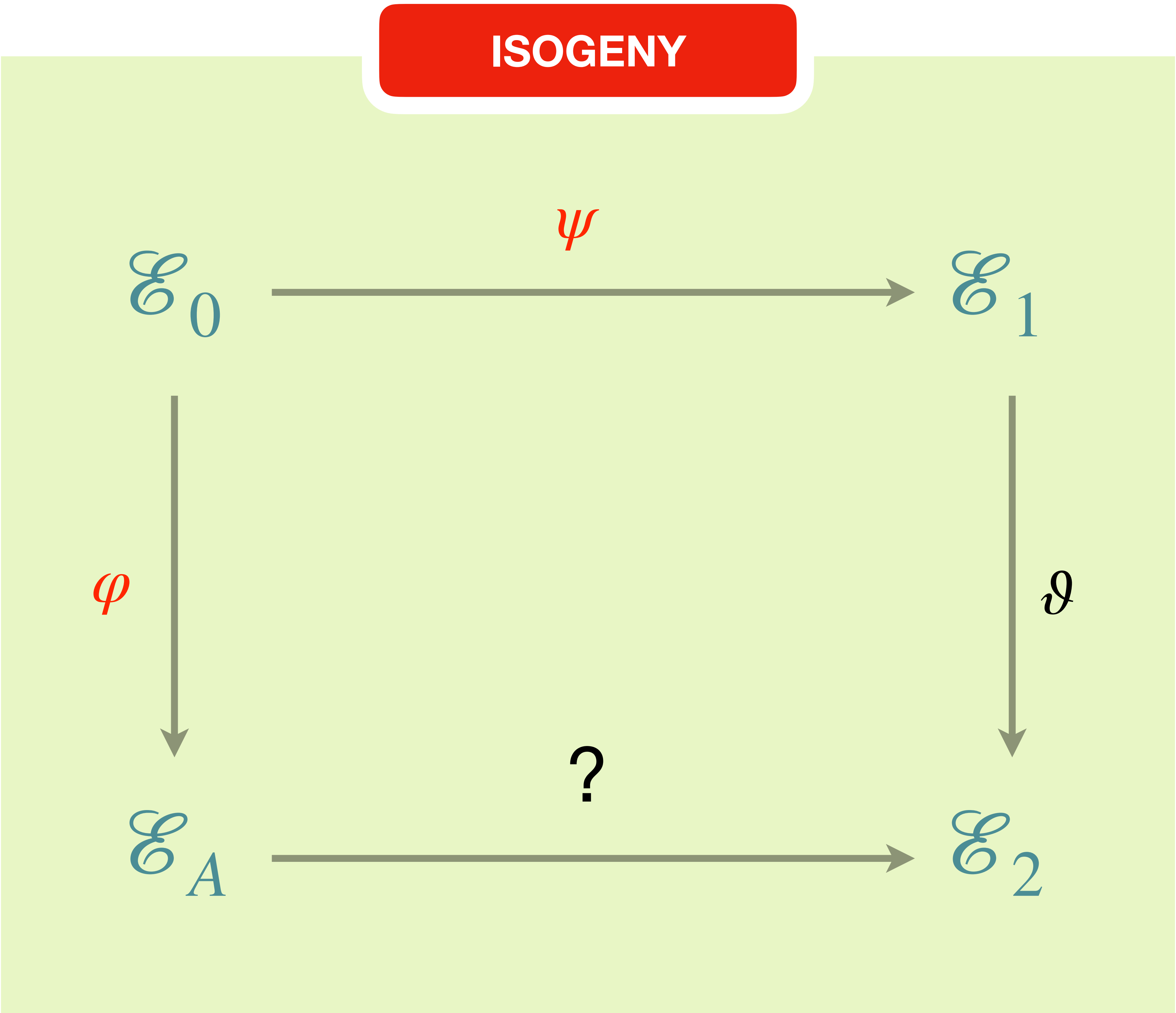

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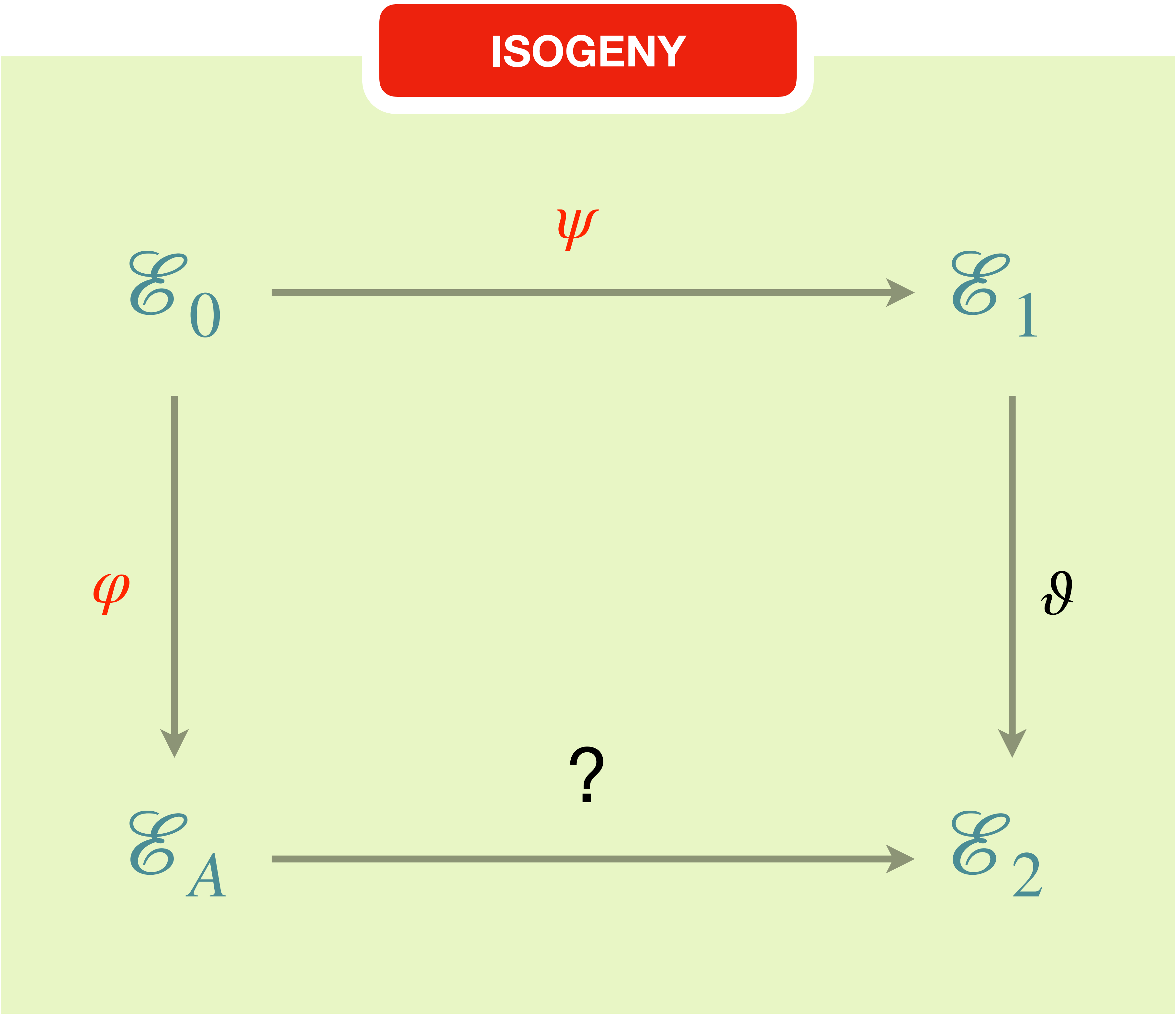

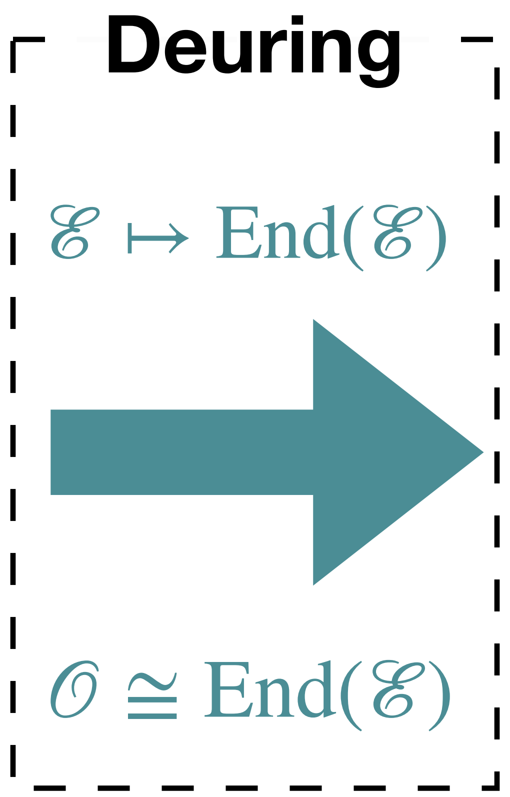

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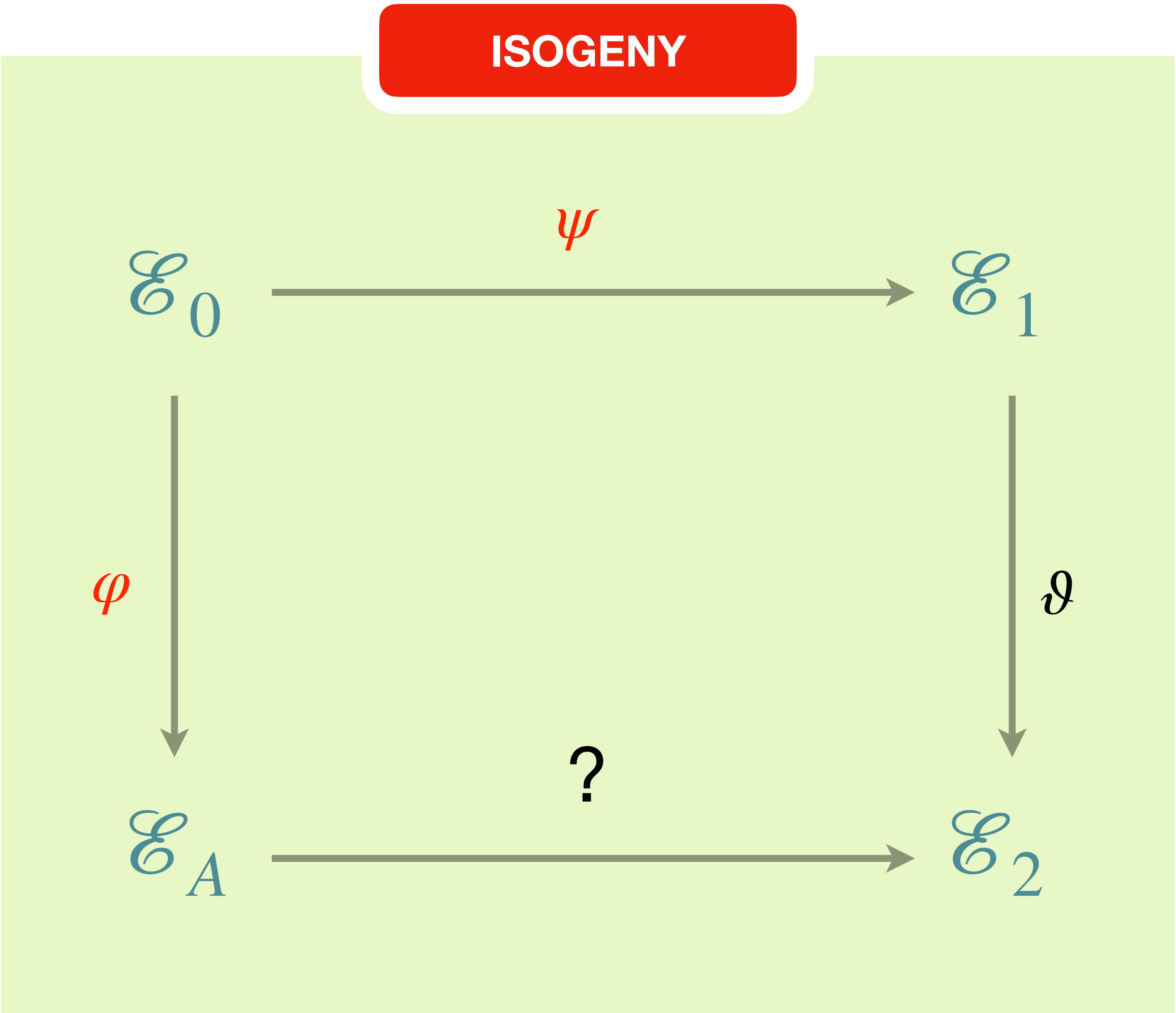

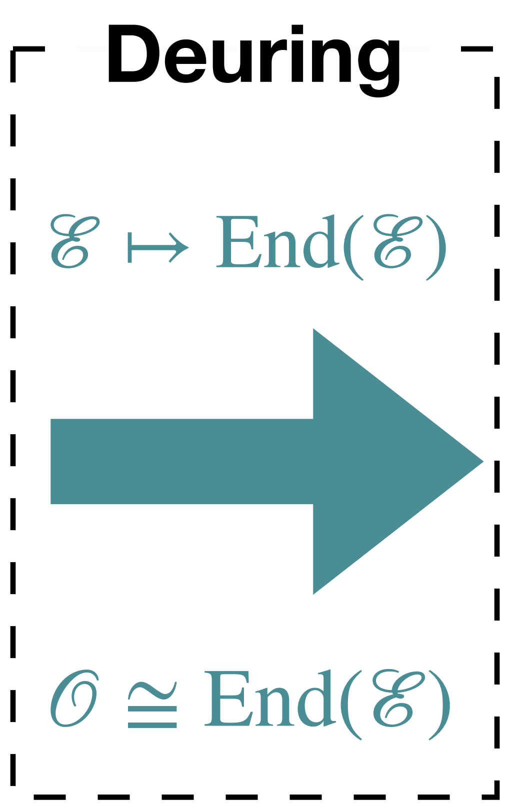

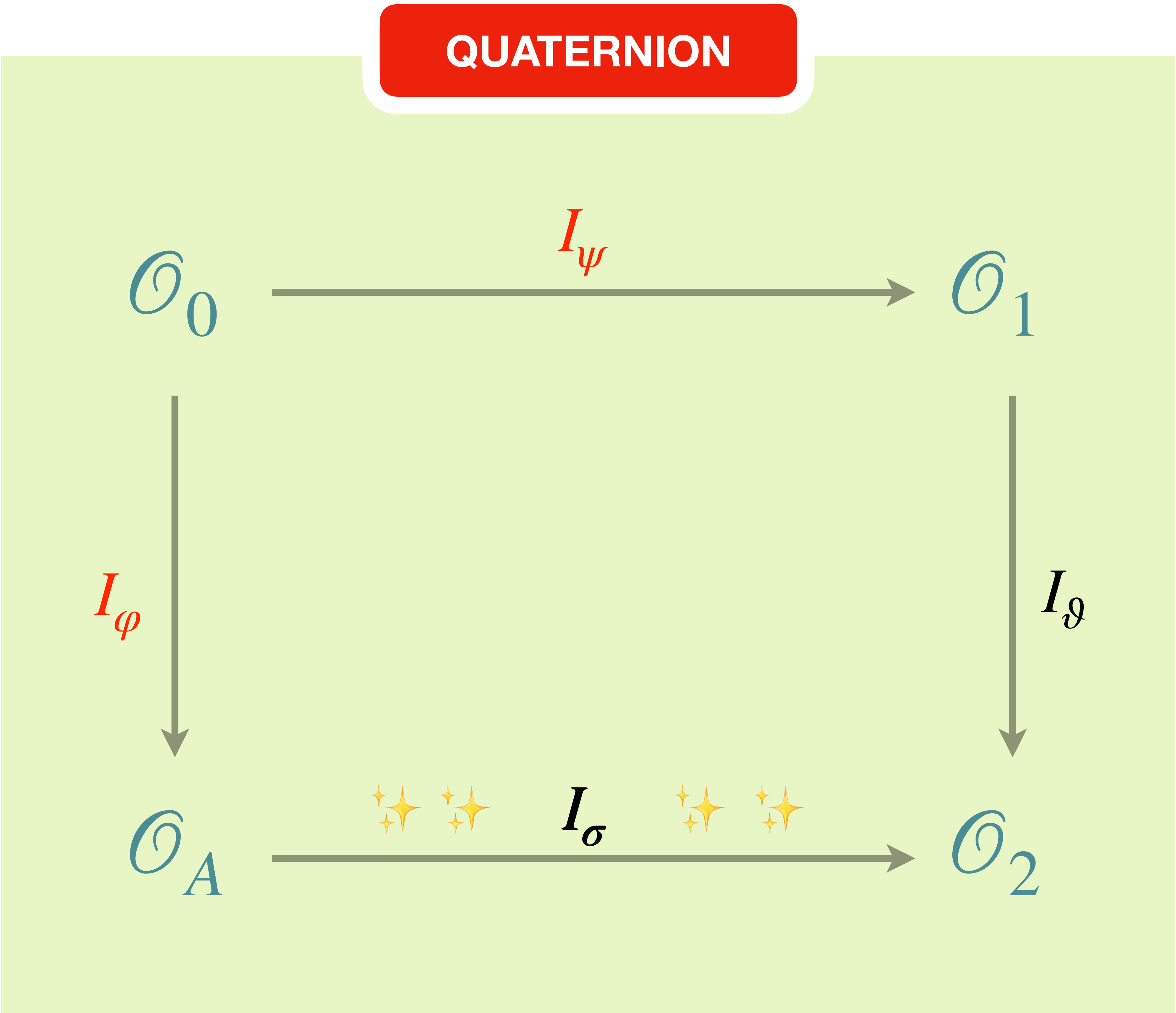

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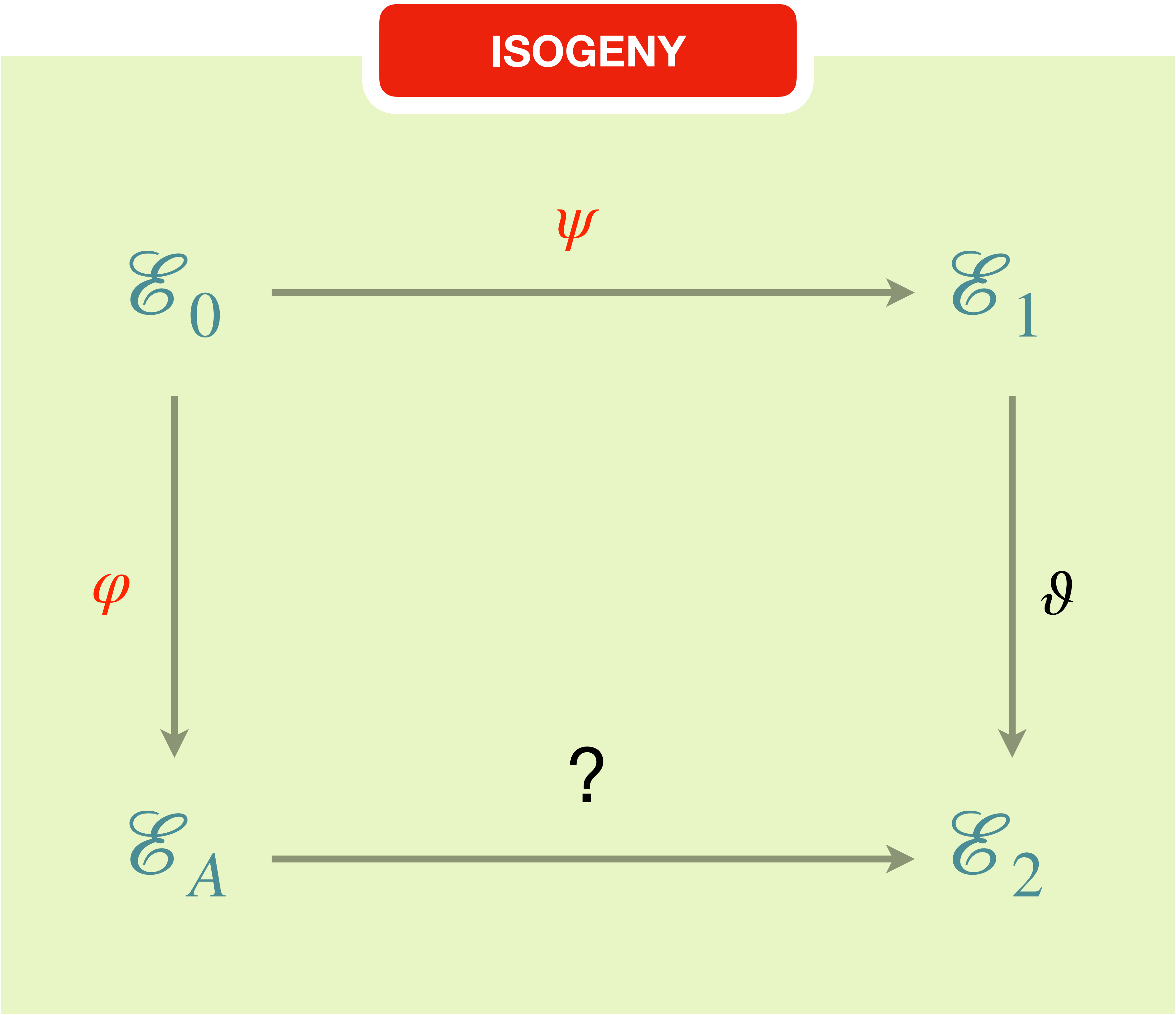
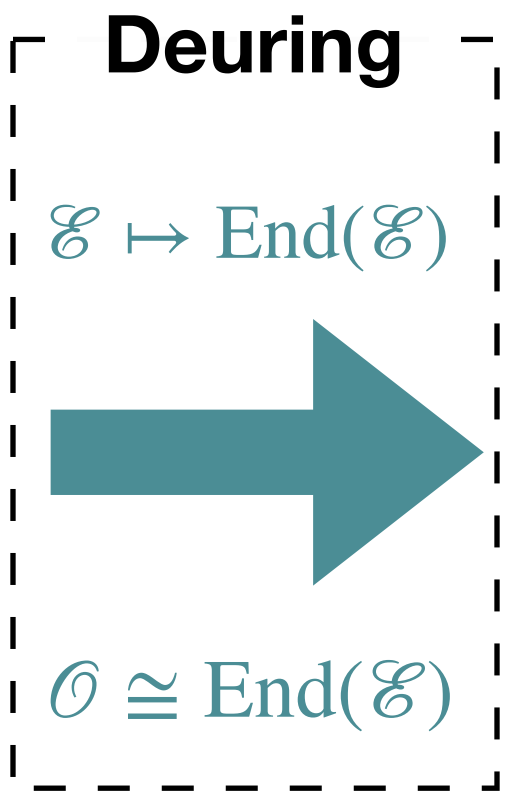
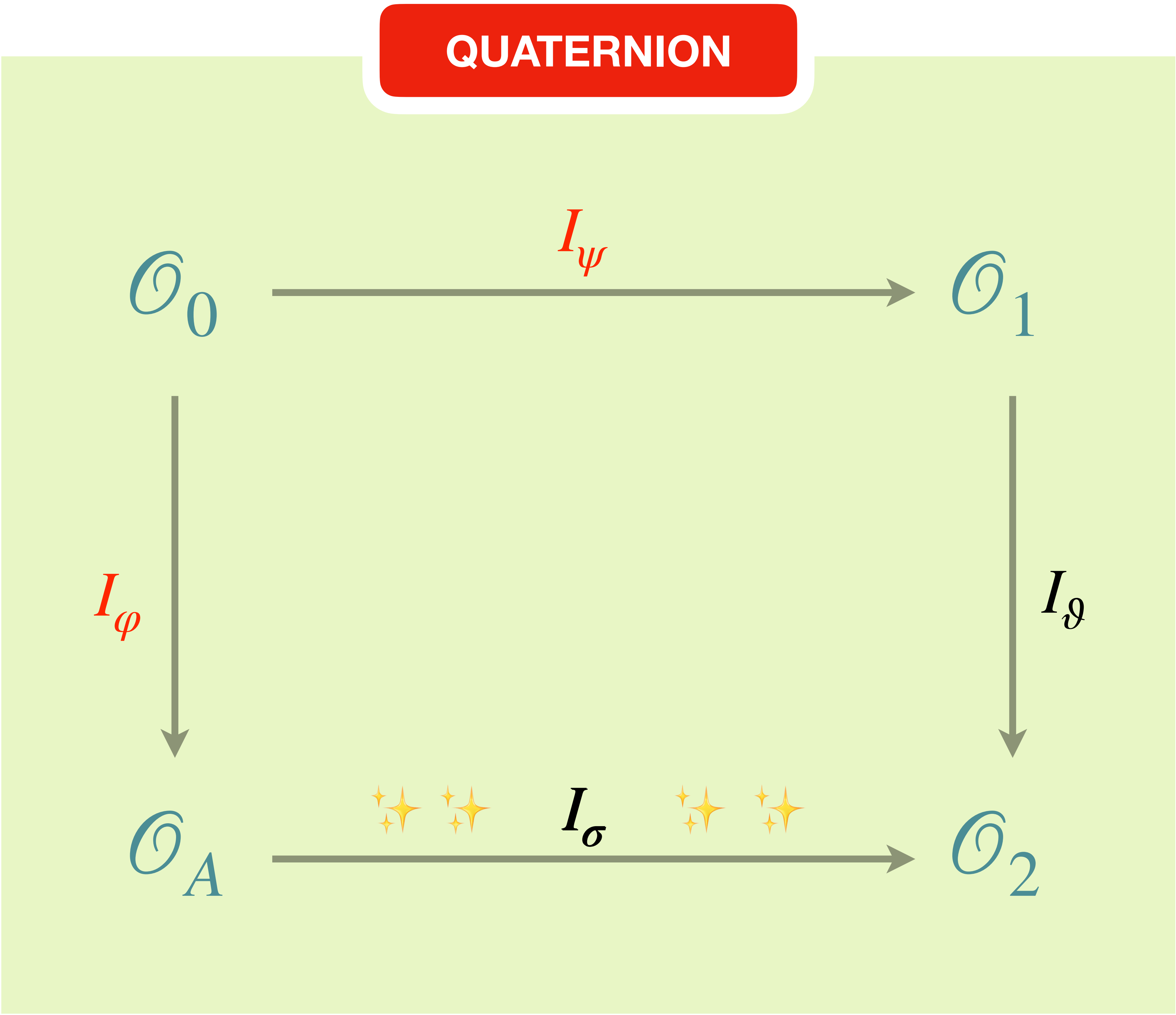




requires knowing $\mathrm{End}(\mathscr{E})$



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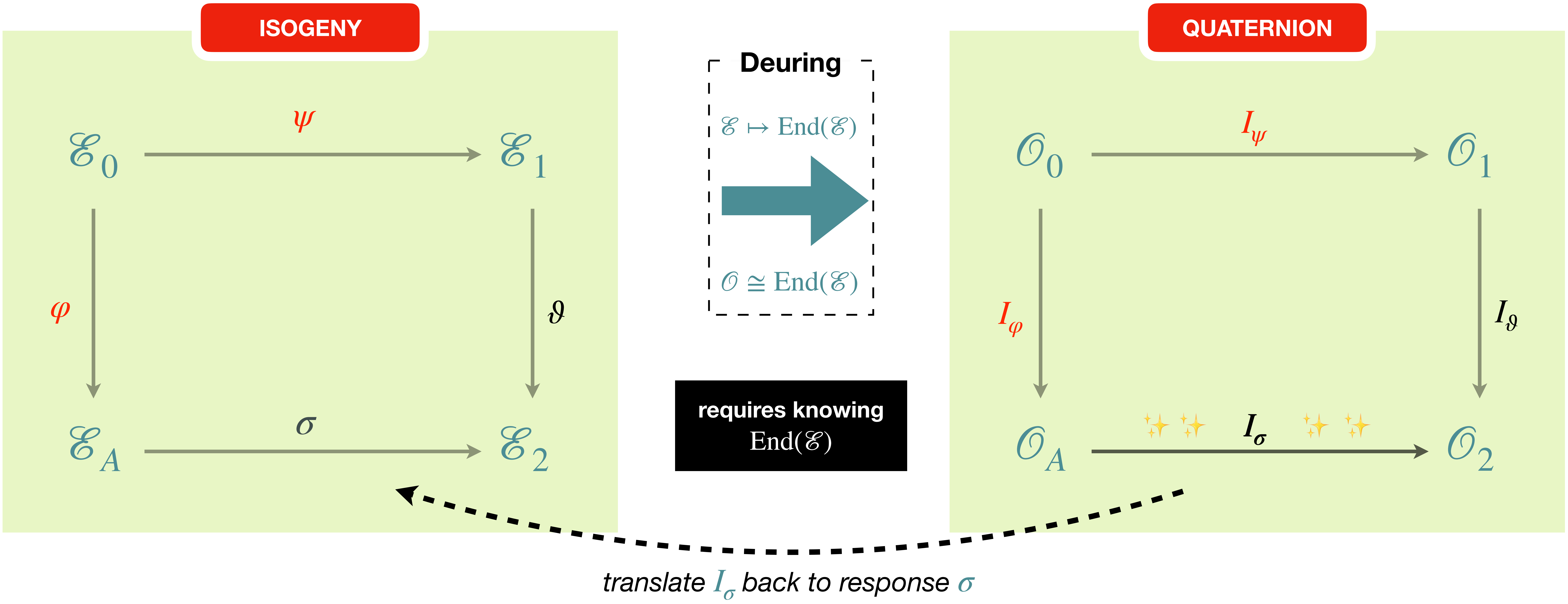
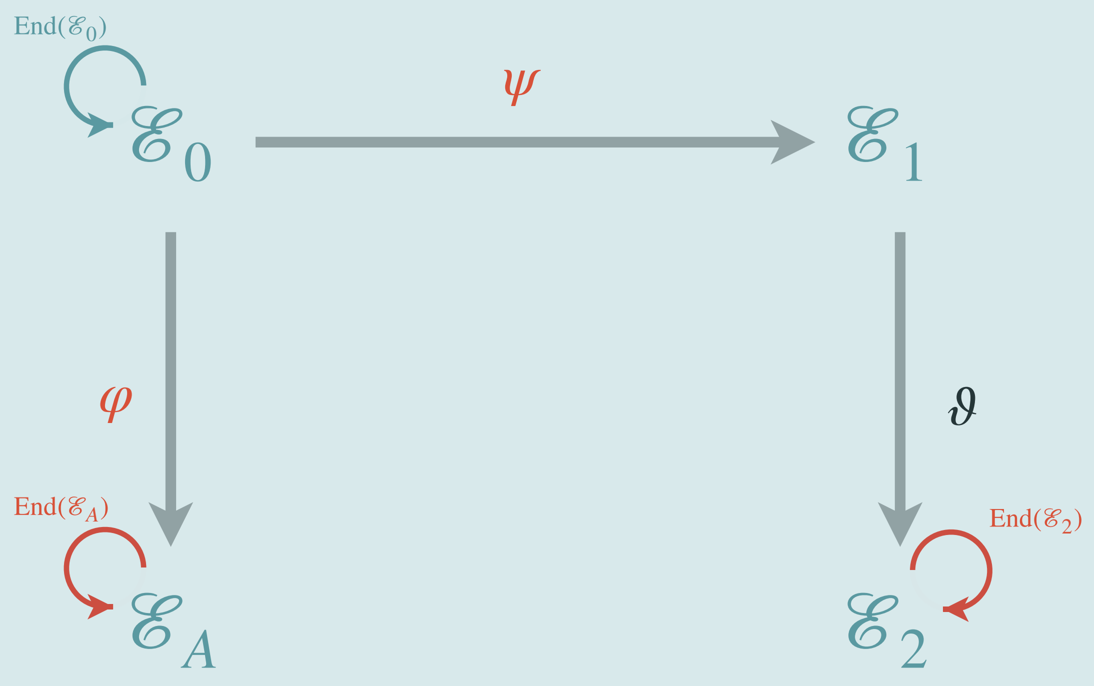


Main recipe for SQIsign: three challenges

0

SETUP

Build the square in the isogeny world, translate to the quaternion world



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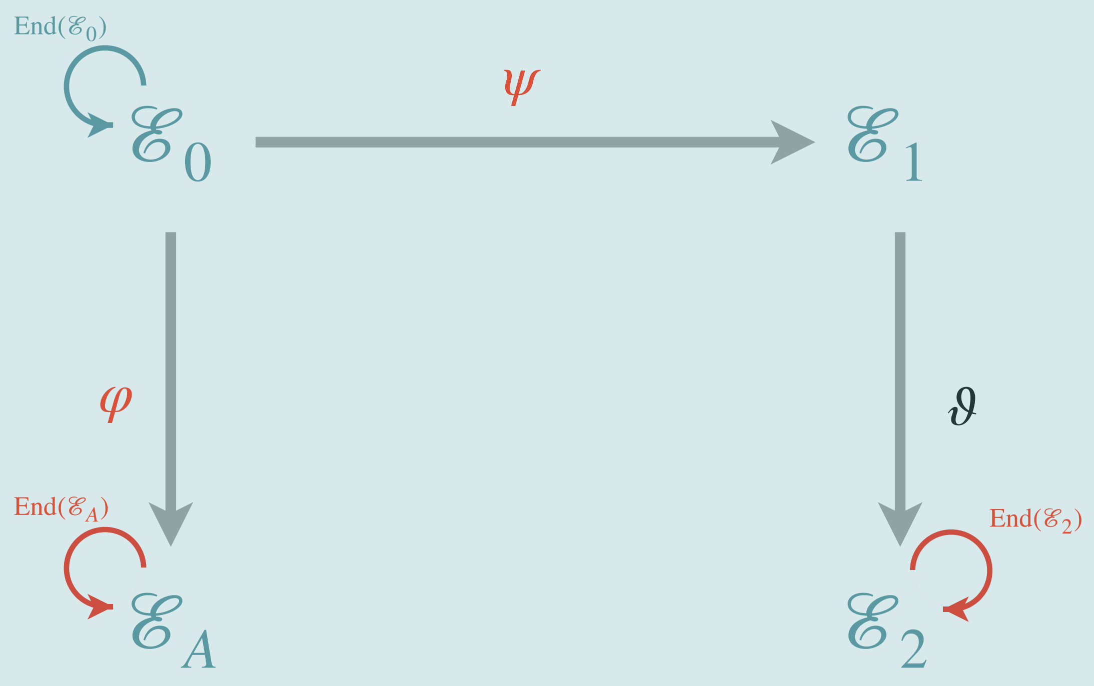

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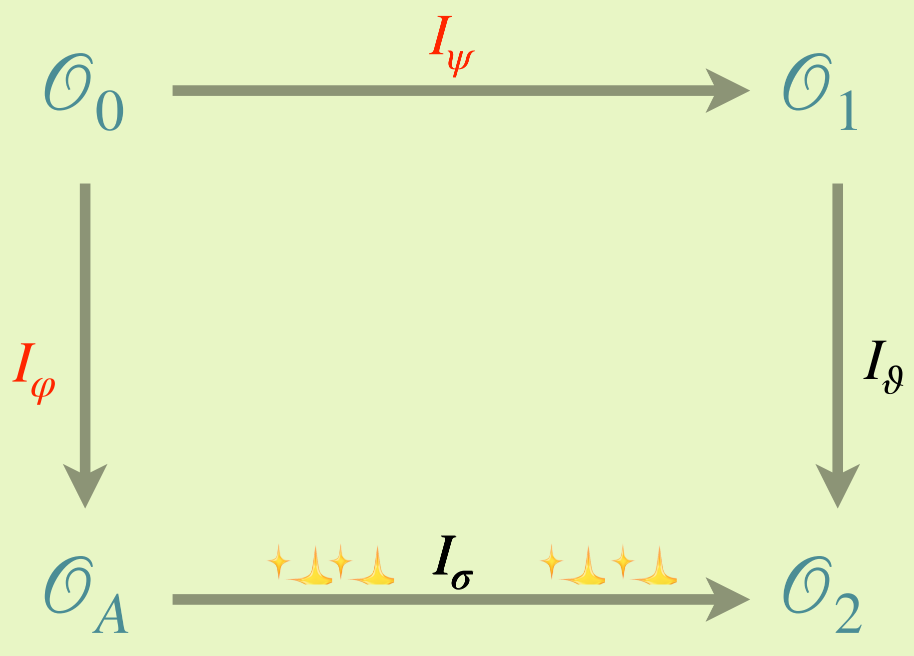
Given the quaternion setup, find the "right" ideal I_σ up to some conditions



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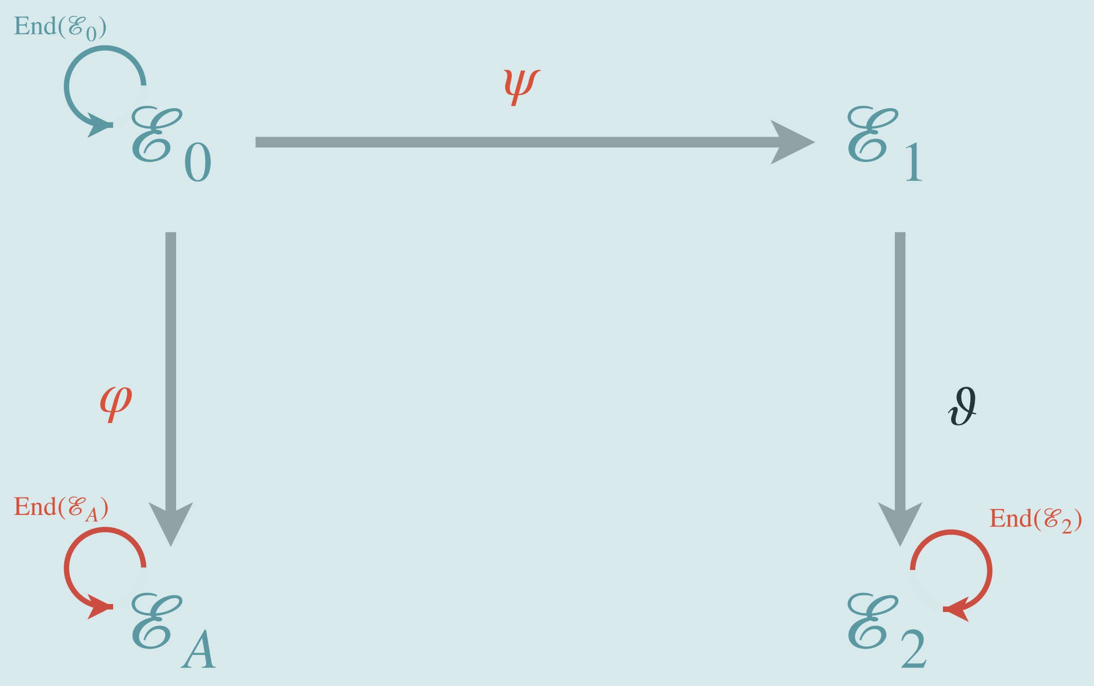

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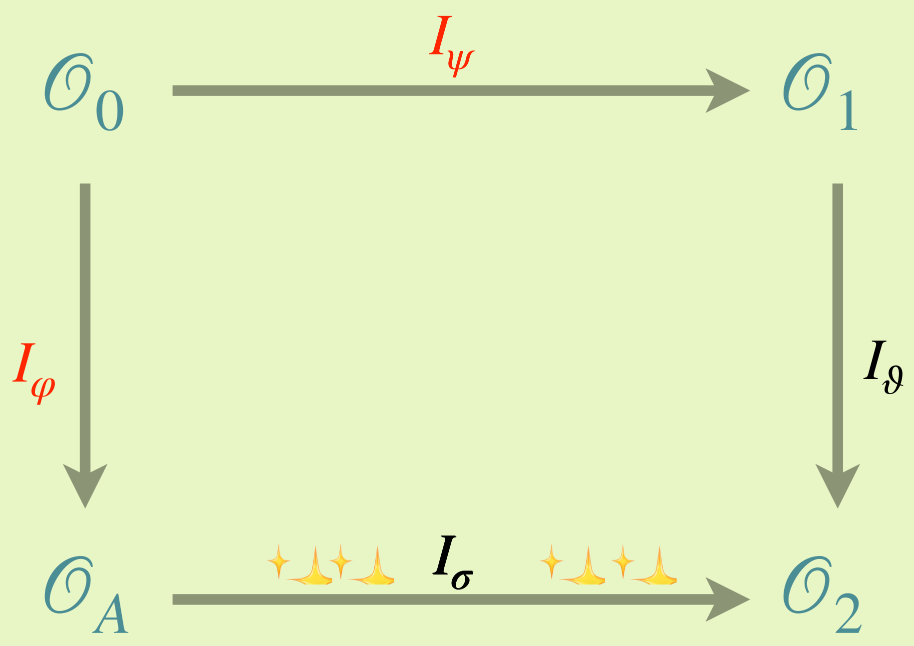
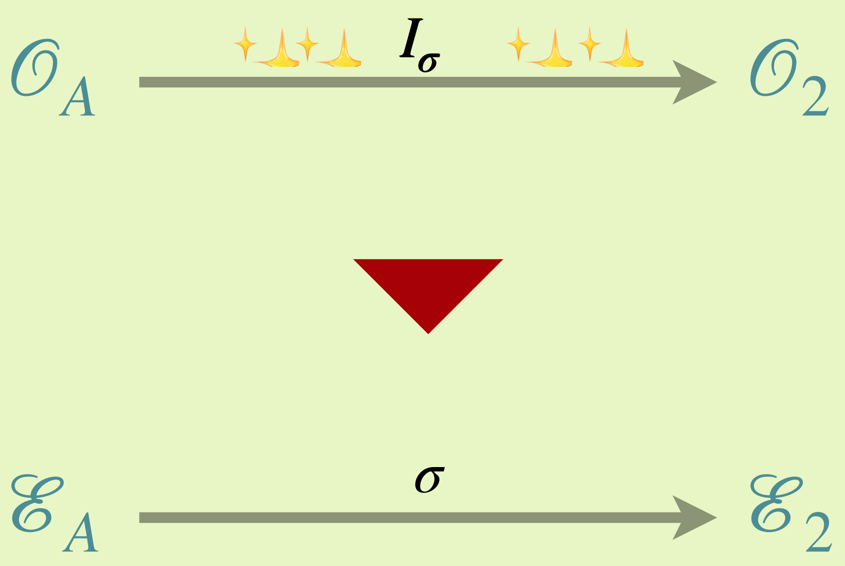


2

IDEAL TO ISOGENY

Given this ideal I_σ, translate back to an isogeny $\sigma : E_A \to E_2$



PART 2
Quaternions!

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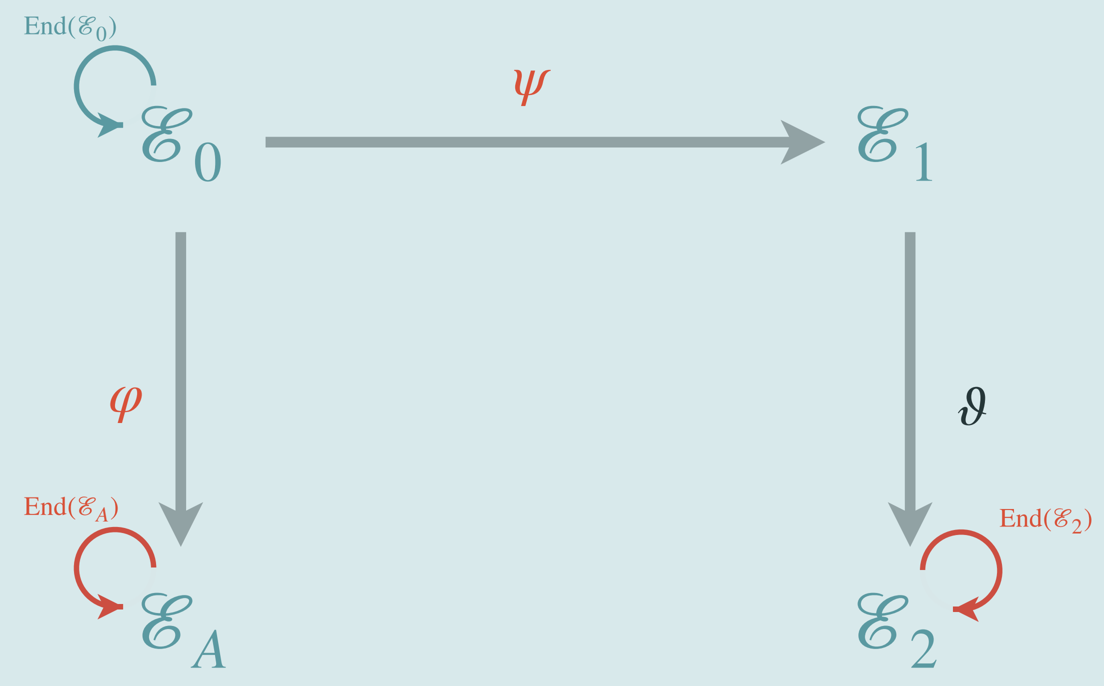

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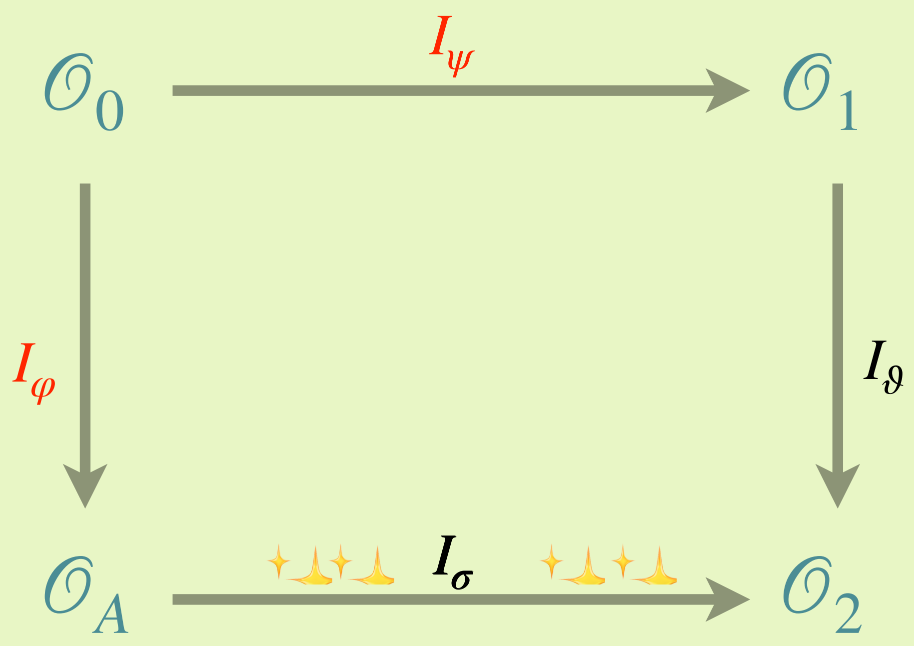

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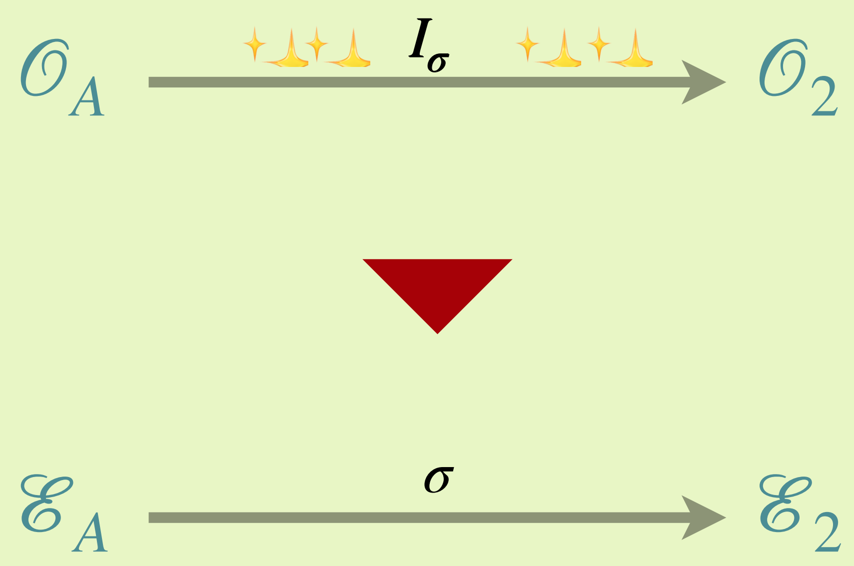

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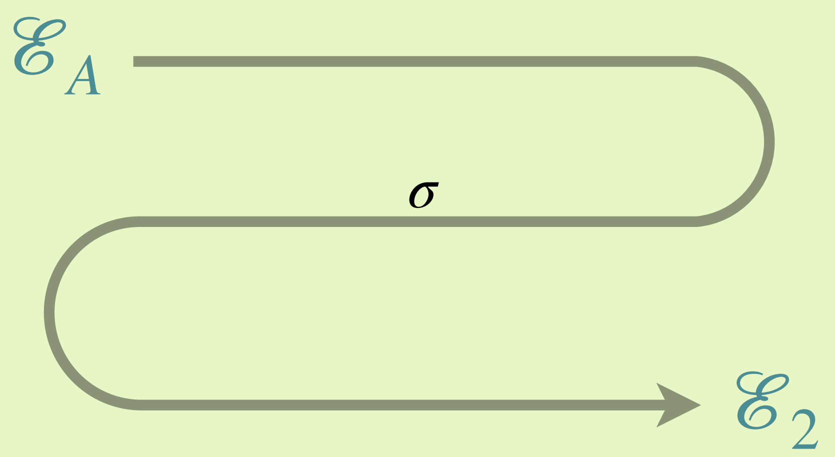
VERIFY

Compute the isogeny σ, which proves knowledge of the secret key φ



(determines verification speed)

PART 2
Quaternions!

Main recipe for SQIsign: three challenges

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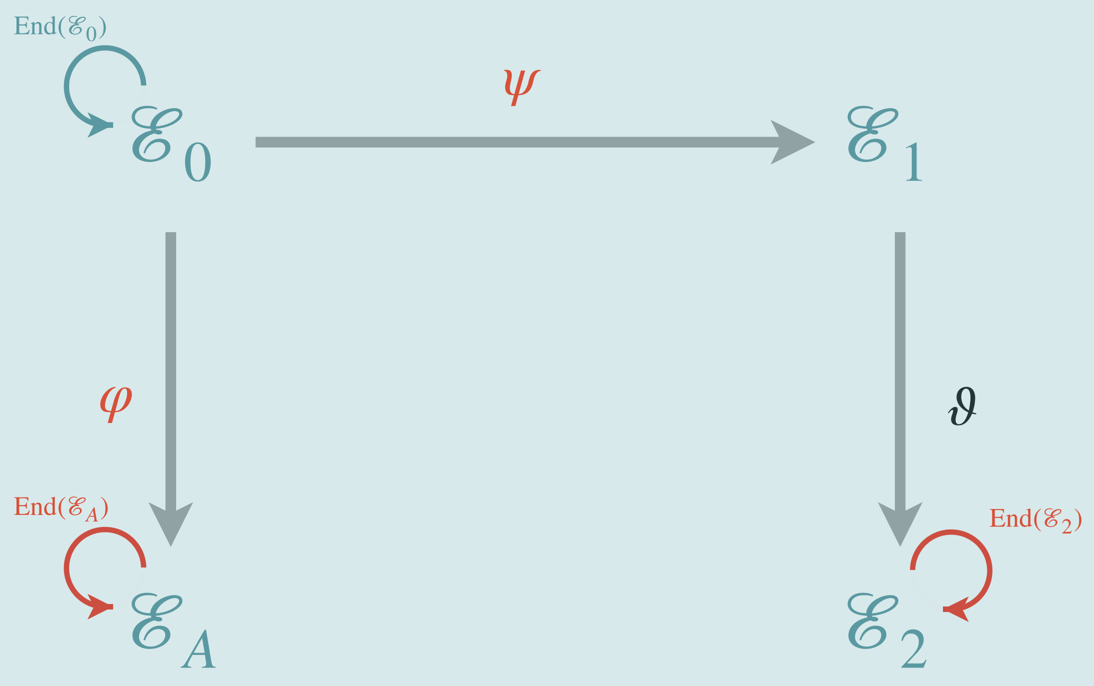
Build the square in the isogeny world, translate to the quaternion world



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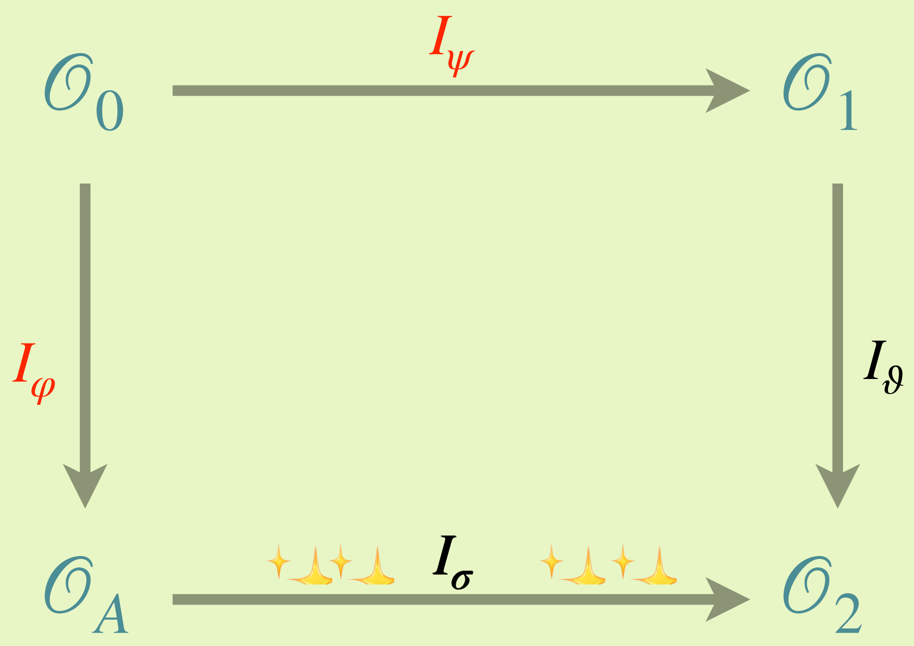

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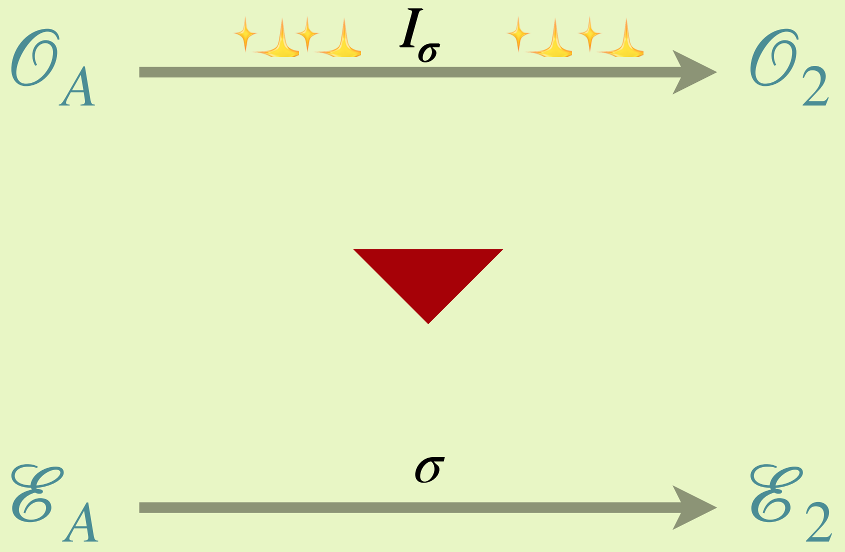
IDEAL TO ISOGENY

Given this ideal I_σ,
translate back to an isogeny
$\sigma : E_A \to E_2$



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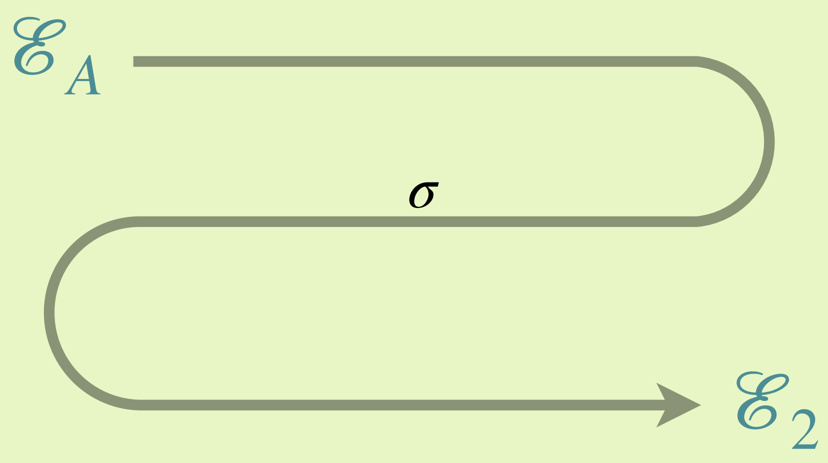

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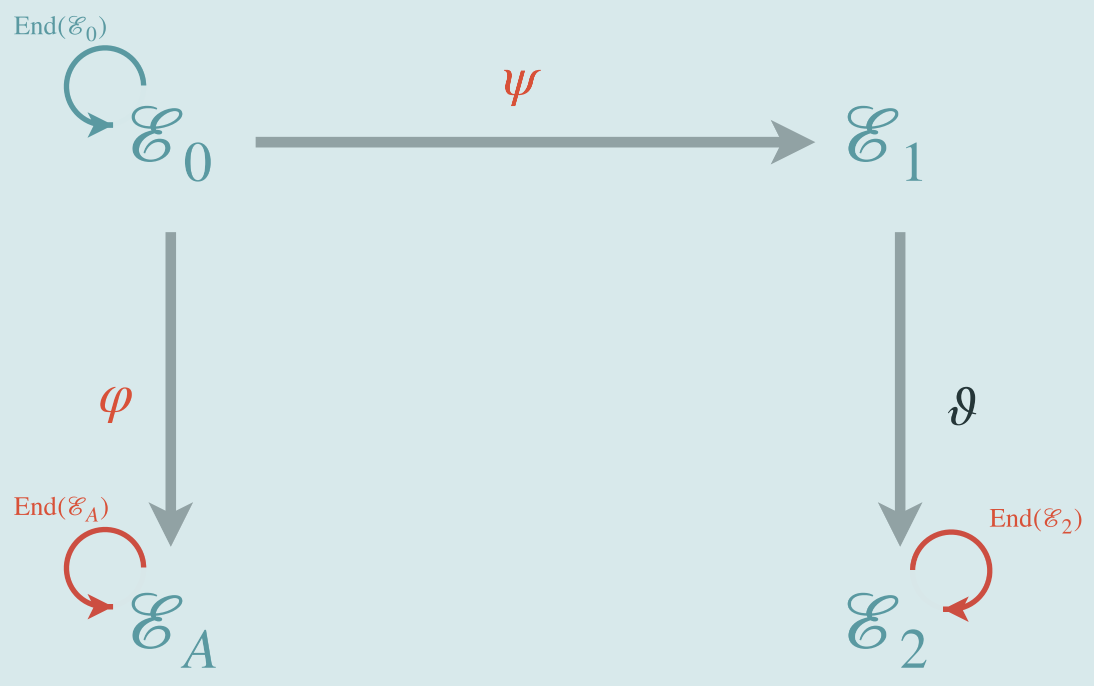

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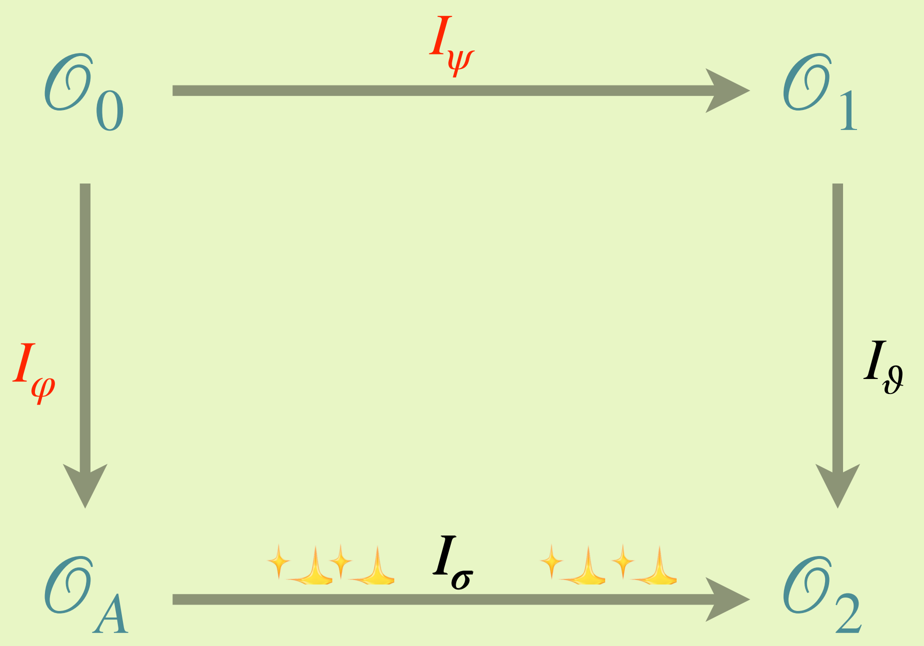

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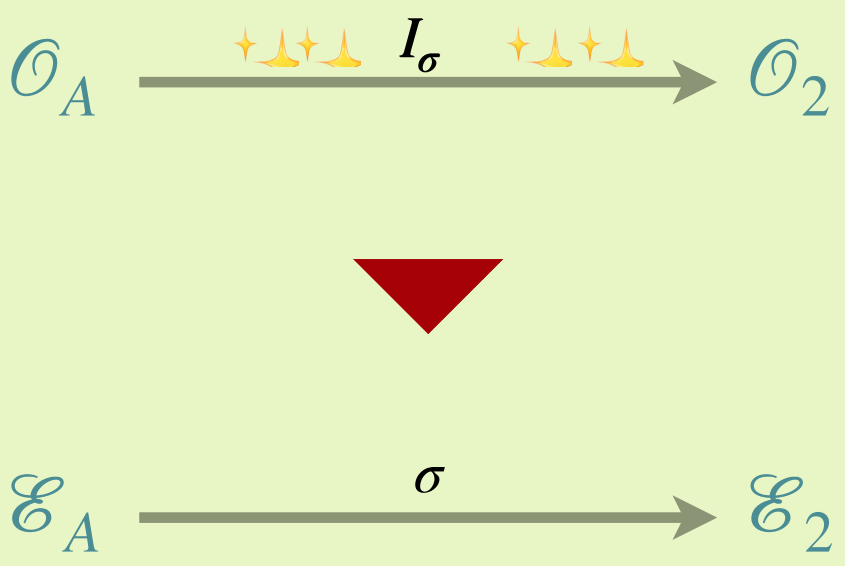
IDEAL TO ISOGENY

Given this idea translate back to an iso y $\sigma : E_A \to E_2$



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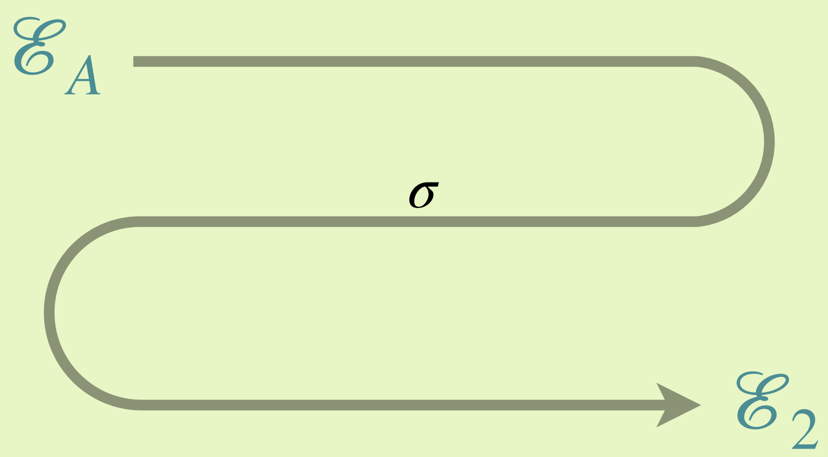


(determines verification speed)

KU LEUVEN

Our plan for today

1

Making the square work…

$$\mathscr{E} \xrightarrow{\varphi} \mathscr{E}'$$

with isogenies!

2

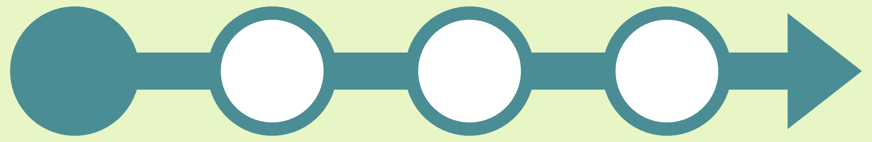
Decomposing the square

$$\mathrm{End}(\mathscr{E}) \xrightarrow{\sim} \mathcal{O}$$

with quaternions!

3

SQIsign, SQIsignHD



SQIsign2D, SQIsignXD…?

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PART 3

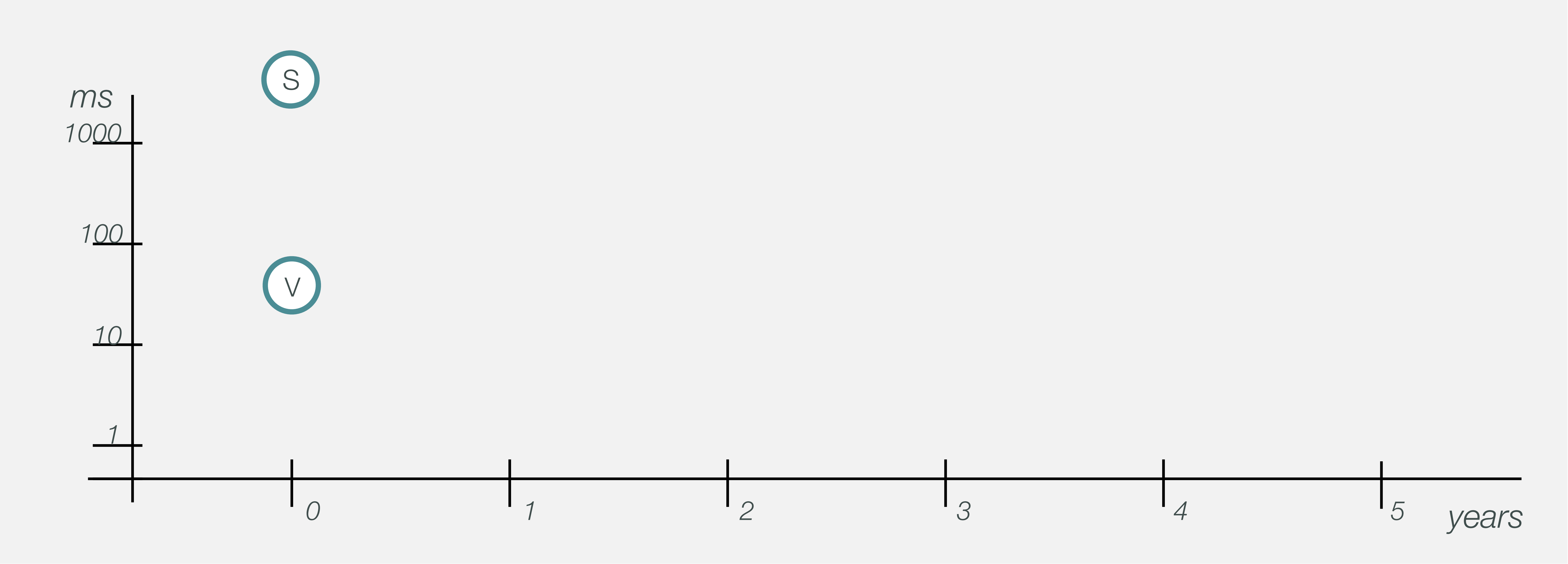
The Variants

SQIsign

A new isogeny-based signature scheme, with **high soundness.**

2020	2022	2023	2024	2025

PART 3
The Variants



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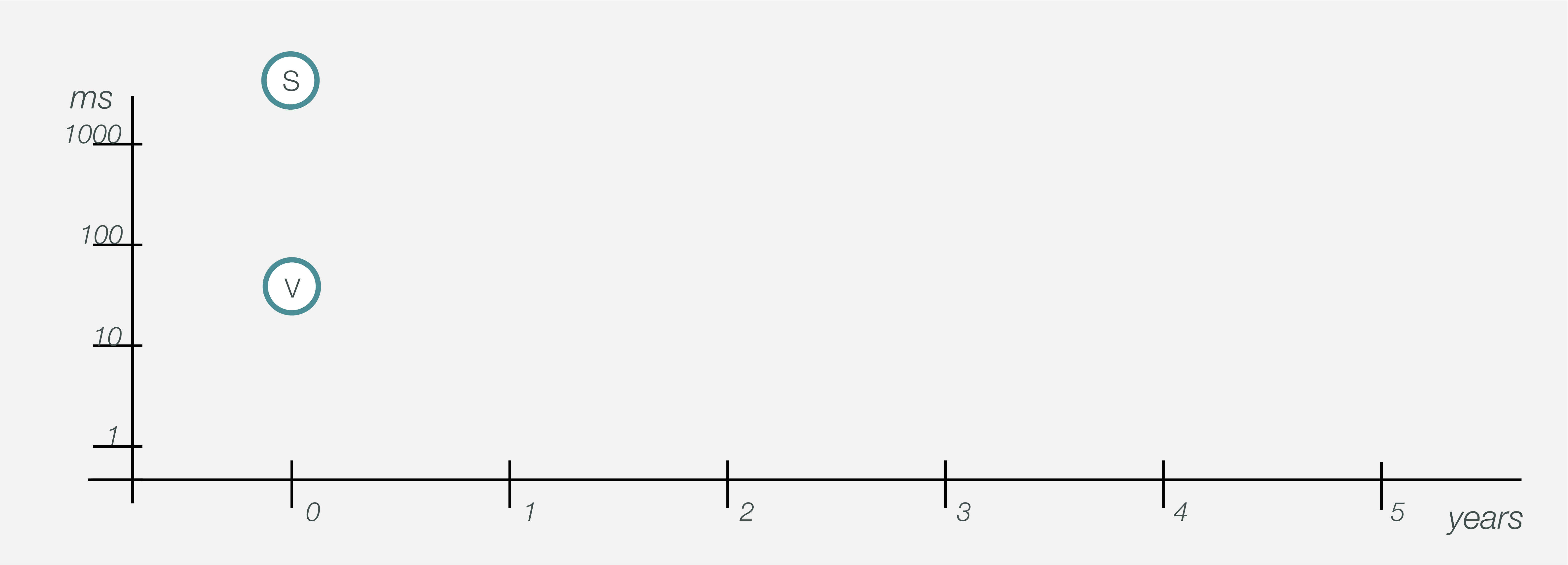

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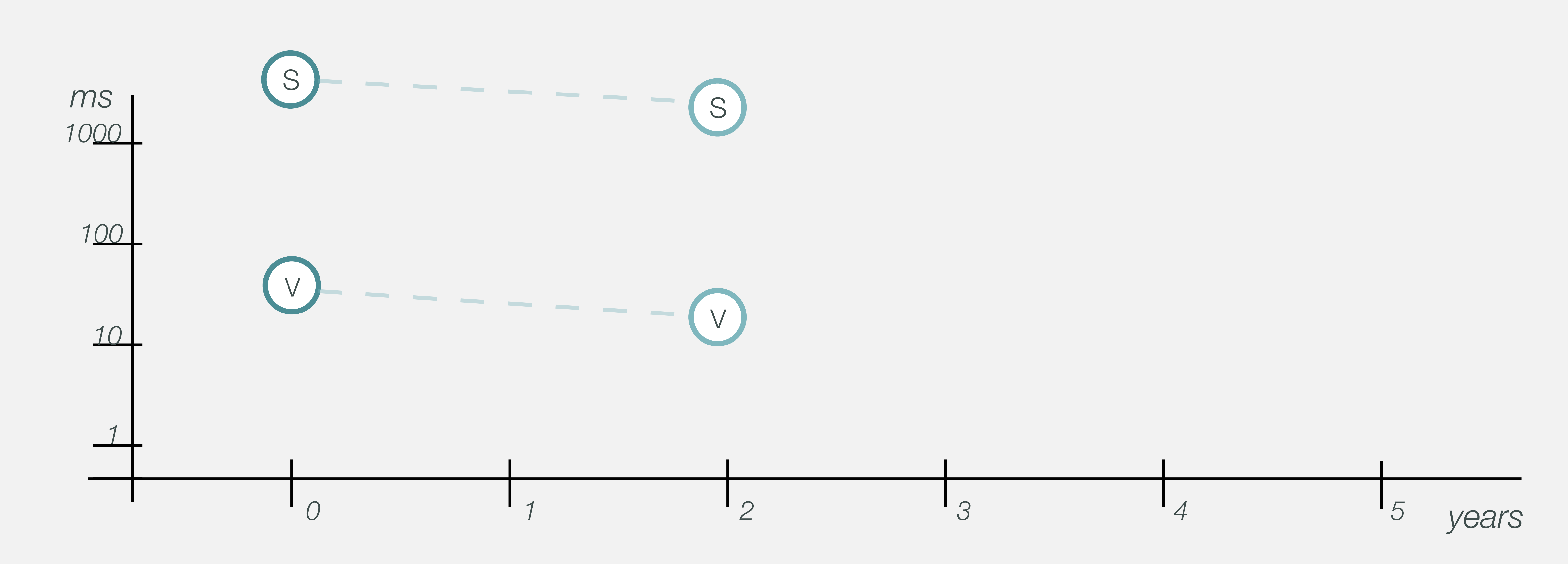
2020 | 2022 | 2023 | 2024 | 2025

summary

1. **Find Ideal**: KLPT (magic)
2. **Id-2-Isog:** Slow, tedious
3. **Verify**: deg. 2^{1000} isogeny

PART 3
The Variants



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SQIsign2

A new algorithm to translate ideals to isogenies.

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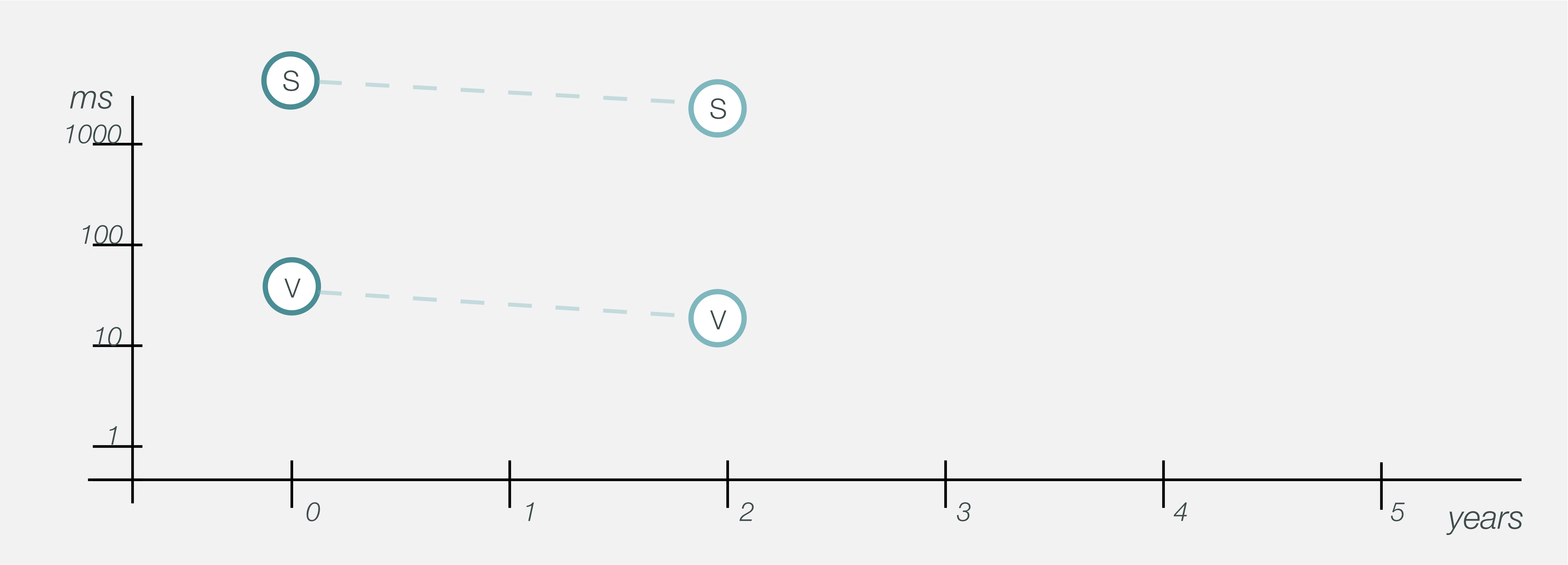

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SIKE breaks

SIKE was destroyed using **HD isogenies** in the summer of 2022.

PART 3

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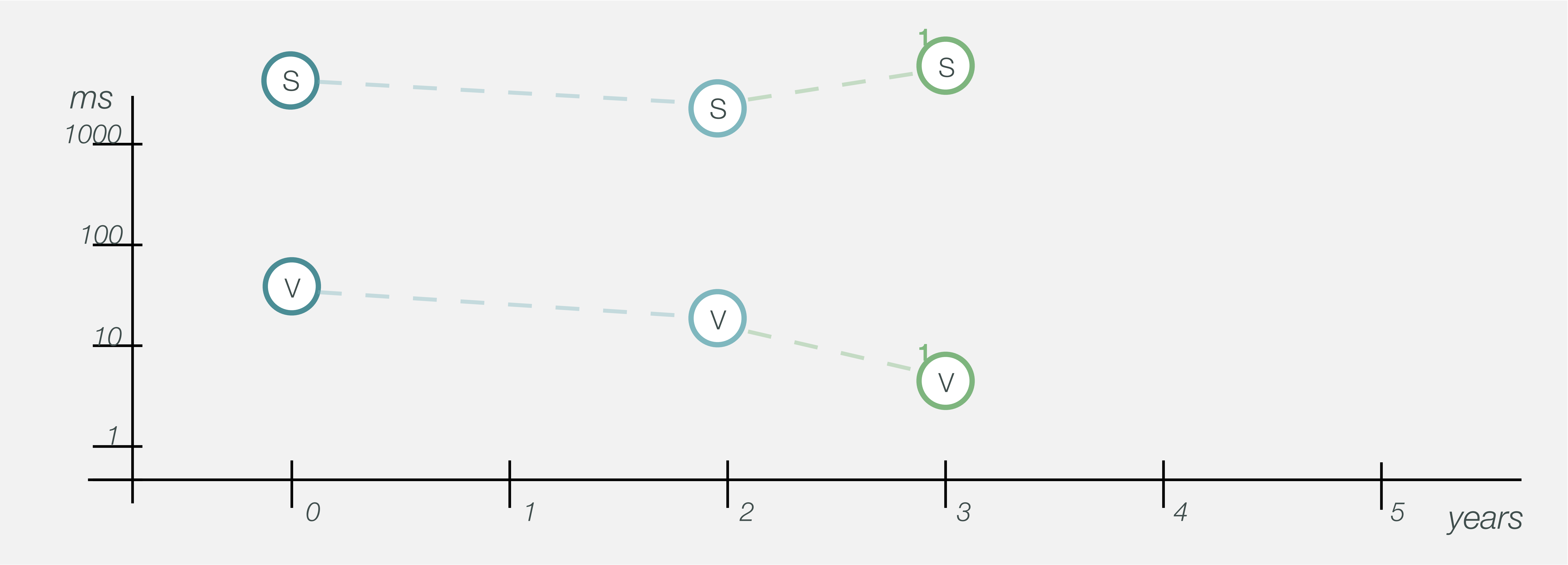

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AprèsSQI

Signing is slow anyway… Push verification to **maximal** efficiency!

2020 | 2022 | 2023 | 2024 | 2025

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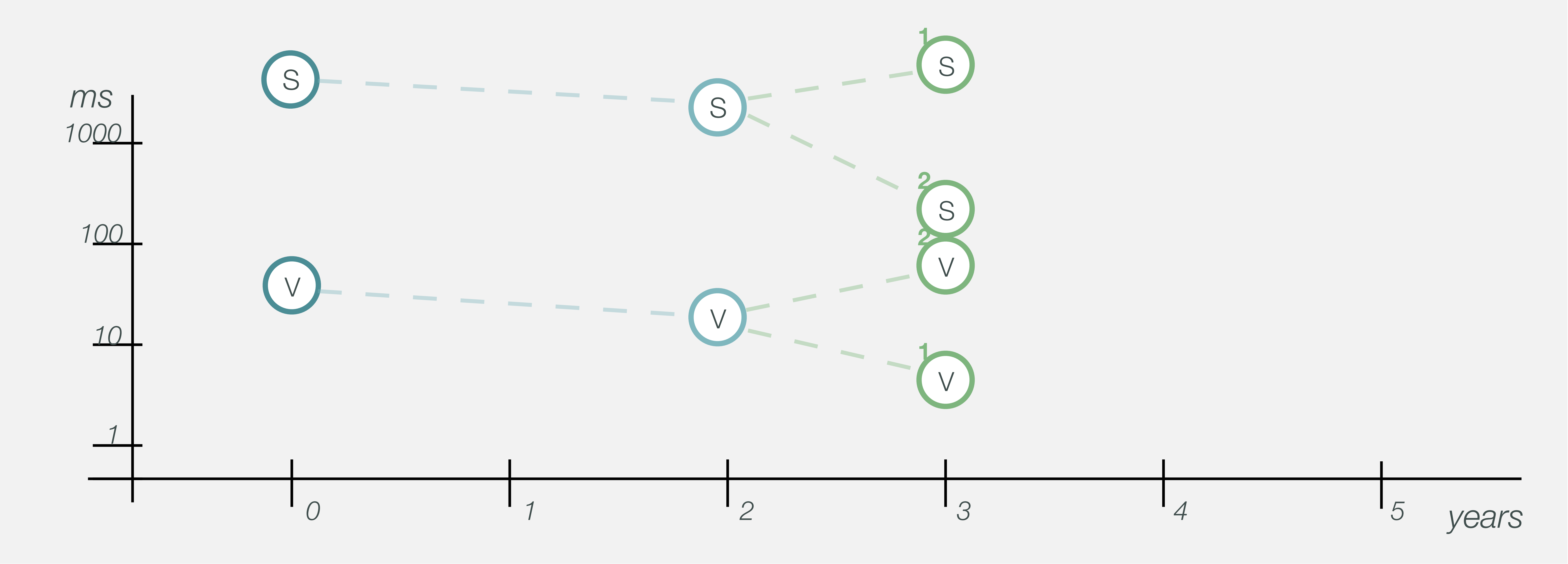
summary

1. **Find Ideal**: KLPT (magic)
2. **Id-2-Isog:** Twice as slow!
3. **Verify**: fast 2^{1000} isogeny

PART 3
The Variants



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2020 | 2022 | 2023 | 2024 | 2025

summary

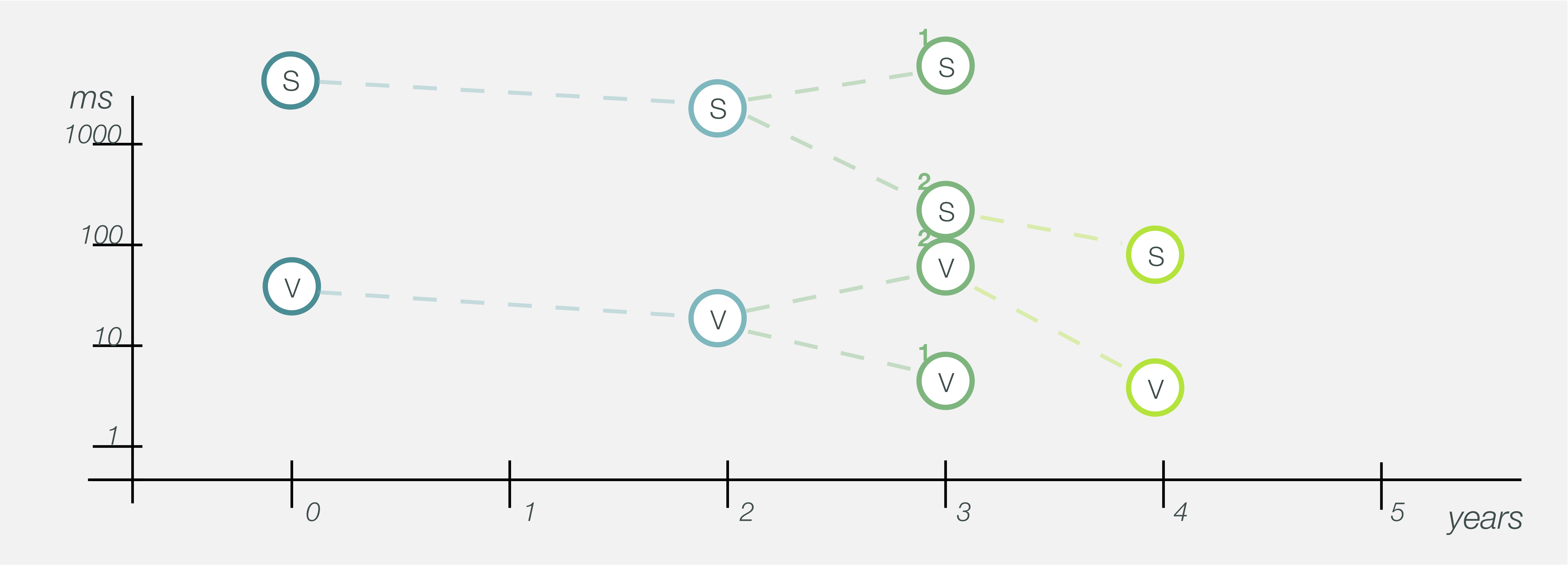
1. **Find Ideal**: HD (easy!)
2. **Id-2-Isog:** Almost trivial
3. **Verify**: SLOW 4D isogeny

SIKE breaks
SIKE was destroyed using **HD isogenies** in the summer of 2022.

SQIsignHD
Represent the response as **HD isogeny**. Requires 4/8-dimensions.

PART 3

The Variants



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SQIsignHD
Represent the response as **HD isogeny**. Requires 4/8-dimensions.

Going 2D
Adapt SQIsignHD to enable verification with **2D isogenies**

summary

1. **Find Ideal**: HD (easy!)
2. **Id-2-Isog:** A few tricks
3. **Verify**: Fast 2D isogeny

PART 3
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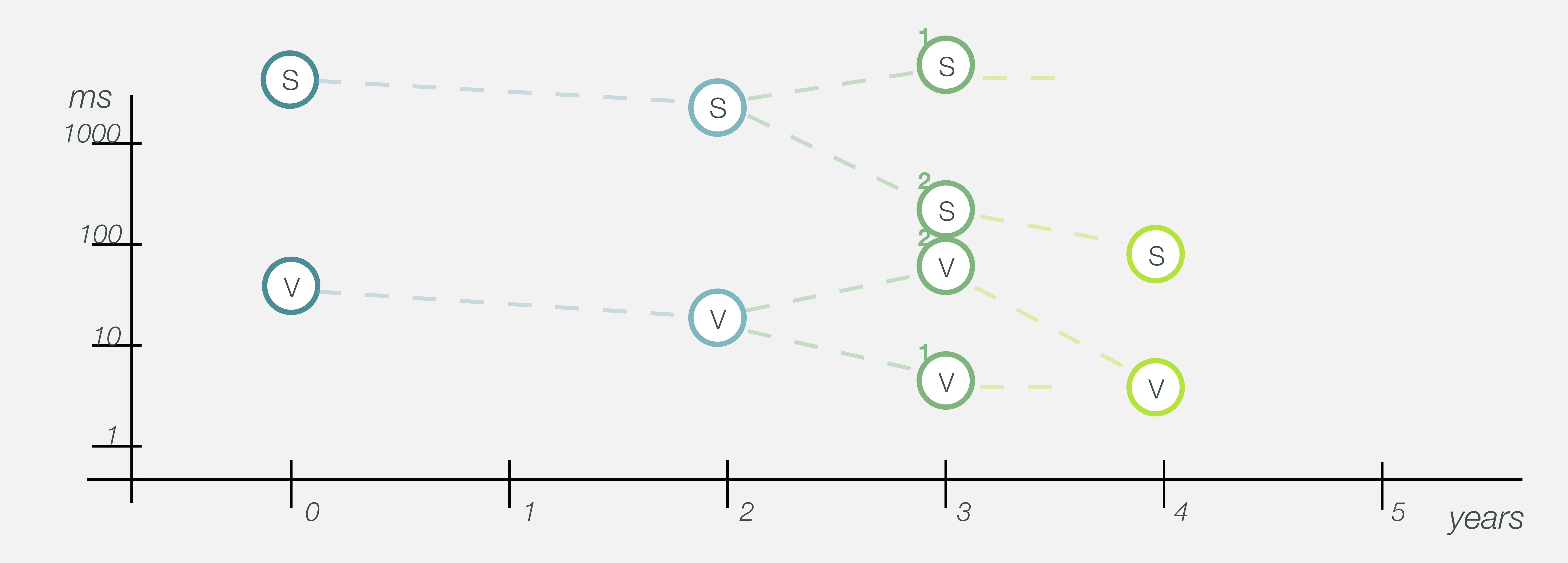

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AprèsSQI
Signing is slow anyway… Push verification to **maximal** efficiency!

What about 1D?
Is there still any use for one-dimensional SQIsign? Should we always do 2D?

2020 | 2022 | 2023 | 2024 | 2025

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SQIsignHD
Represent the response as **HD isogeny**. Requires 4/8-dimensions.

Going 2D
Adapt SQIsignHD to enable verification with **2D isogenies**

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2. **Id-2-Isog:** A few tricks
3. **Verify**: Fast 2D isogeny


SQIsign2DPush: Faster Signature Scheme Using 2-Dimensional Isogenies
Kohei Nakagawa1 and Hiroshi Onuki2[0000-0002-0202-8918]
1 NTT Social Informatics Laboratories, Japan kohei.nakagawa@ntt.com
2 The University of Tokyo, Japan hiroshi-onuki@g.ecc.u-tokyo.ac.jp
Abstract. Isogeny-based cryptography is cryptographic schemes whose security is based on the hardness of a mathematical problem called the isogeny problem, and is attracting attention as one of the candidates for post-quantum cryptography. A representative isogeny-based cryptography is the signature scheme called SQIsign, which was submitted to the NIST PQC standardization competition for additional signature. SQIsign has attracted much attention because of its very short signature and key size among candidates for the NIST PQC standardization. Recently, many new signature schemes using high-dimensional isogenies have been proposed, such as: SQIsignHD, SQIsign2D-West, SQIsign2D-East, and SQIPrime. Last year, SQIsign advanced to Round 2 of the NIST competition and was updated to version 2.0 (we call it SQIsign-v2.0), which is based on SQIsign2D-West. SQIsign-v2.0 achieves smaller signature sizes and faster verification. However, the signing costs
SQIsign2D²: New SQIsign2
Leveraging Power Smooth Isog
Dimension One
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Abstract. In this paper, we propose SQIsign2D², a novel digital signature scheme within the SQIsign2D family. Unlike other SQIsign2D variants, SQIsign2D² employs the prime p = CD − 1 as the field characteristic, where D = 2^e2, C = 3^e3 and C ≈ D ≈ √p. By leveraging accessible C-isogenies, SQIsign2D² significantly reduces the degree requirements for two-dimensional isogeny computations, thereby lowering the overall computational overhead compared to other SQIsign2D
SQIPrime: A dim
with non-smooth
Max Duparc and Tako Boris Fouotsa
EPFL, Lausanne, Switzerland
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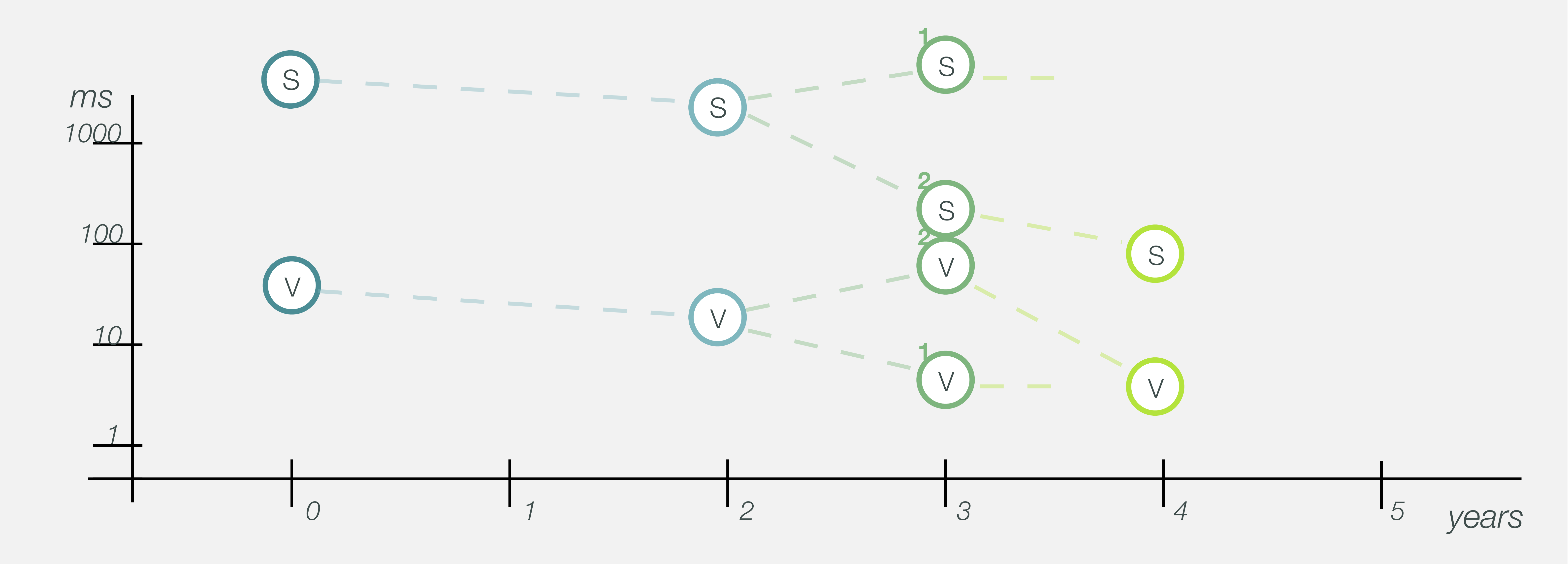
Abstract. We introduce
the Fast, the Small, and the Safer
Andrea Basso1,2[0000-0002-3270-1069], Pierrick Dartois3,4[0009-0008-2808-9867], Luca De Feo2[0000-0002-9321-0773], Antonin Leroux5,6[0009-0002-3737-0075], Luciano Maino1[0009-0005-4495-5102], Giacomo Pope1,7, Damien 3,4[0000-0003-4378-4274], and Benjamin Wesolowski8[0000-0003-1249-6077]
University of Bristol, Bristol, United Kingdom
Zürich, Switzerland
IMB, UMR 5251, F-33400 Talence, France
F-33400, Talence, France

PART 3
The Variants



SQIsign
A new isogeny-based signature scheme, with **high soundness.**

SQIsign2
A new algorithm to translate ideals to isogenies.

AprèsSQI
Signing is slow anyway… Push verification to **maximal** efficiency!

What about 1D?
Is there still any use for one-dimensional SQIsign? Should we always do 2D?

The BOOM
Everybody makes their own version of SQIsign? What is SQIsign?

2020 — 2022 — 2023 — 2024 — 2025

summary

1. **Find Ideal**: HD (easy!)
2. **Id-2-Isog:** A few tricks
3. **Verify**: Fast 2D isogeny

SIKE breaks
SIKE was destroyed using **HD isogenies** in the summer of 2022.

SQIsignHD
Represent the response as **HD isogeny**. Requires 4/8-dimensions.

Going 2D
Adapt SQIsignHD to enable verification with **2D isogenies**

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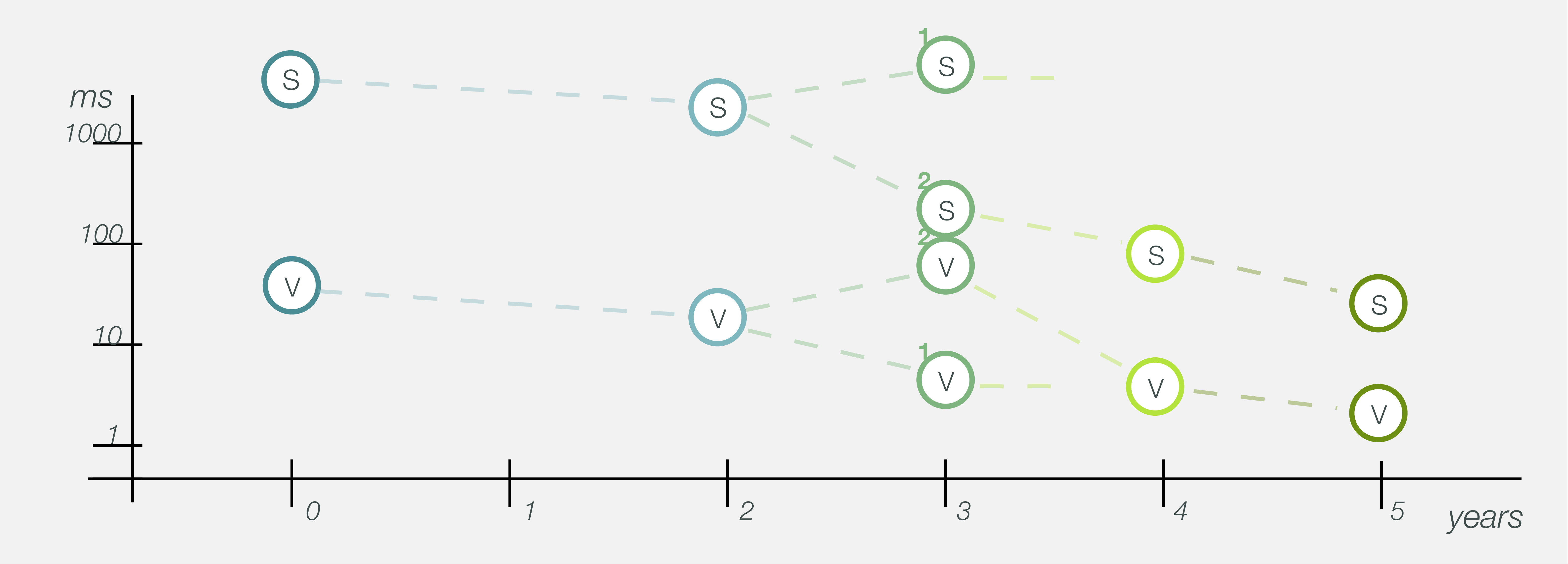

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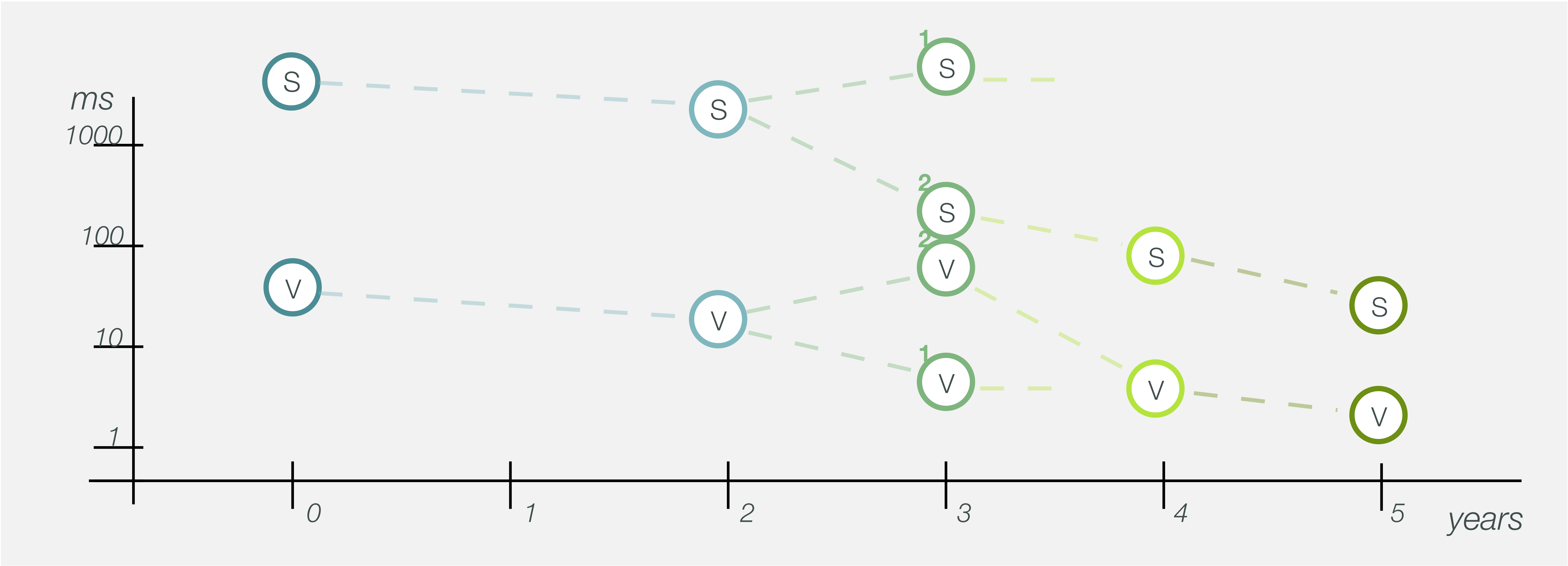

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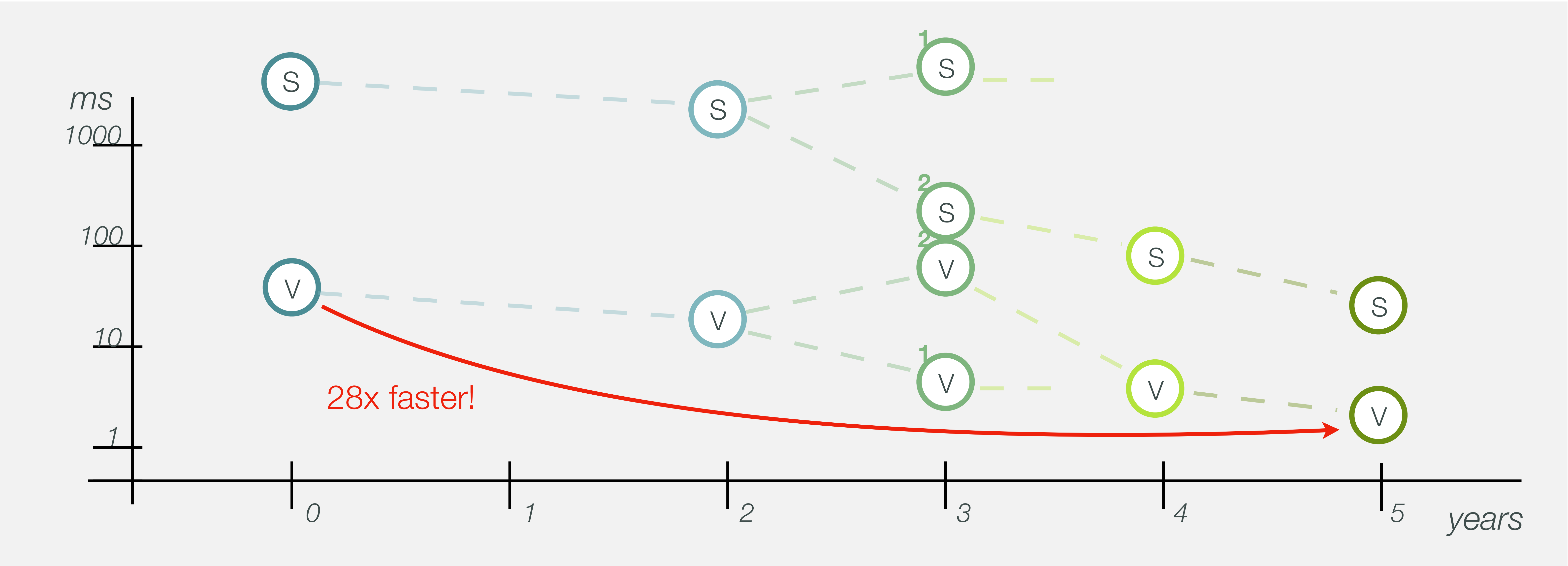

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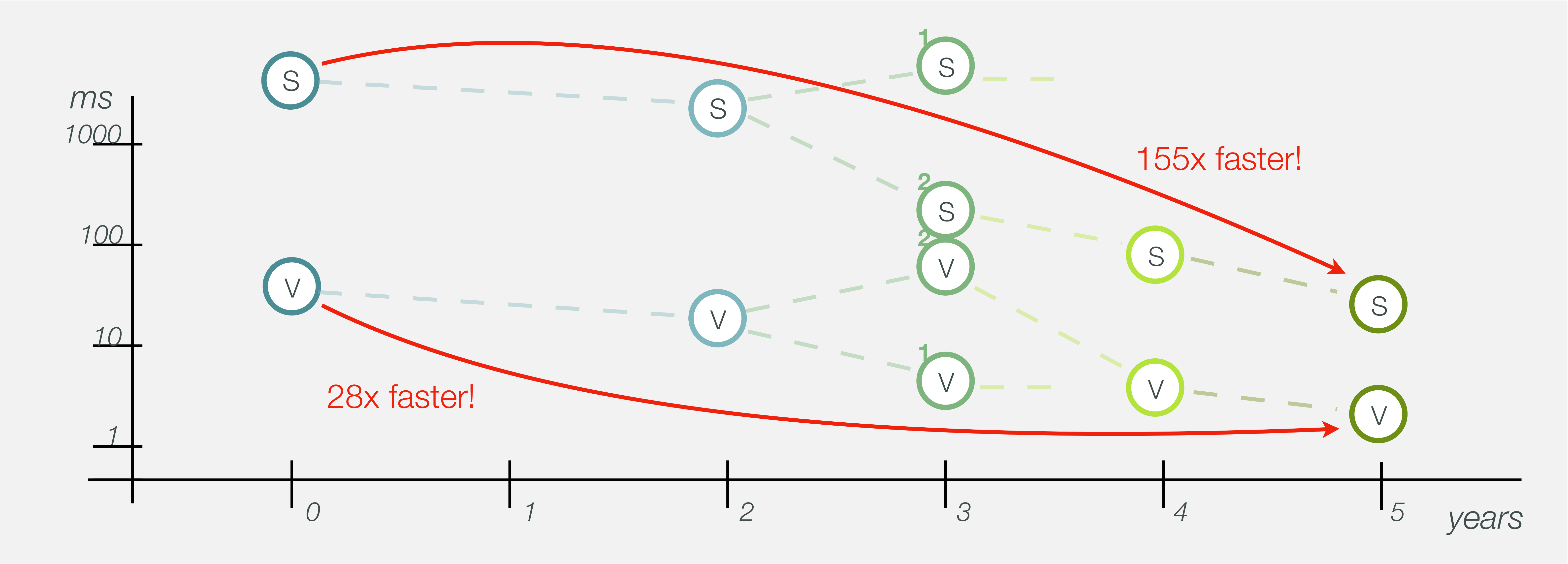

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